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SHORTAGE IN A MULTI-ITEM INVENTORY
SYSTEM WITH A CONSTRAINT FOR
SINGLE PERIOD.

BY

Park, Hwa Jin

December 1988

Thesis Advisor Dan Trietsch

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A Marginal Analysis for Minimizing Shortage in A Multi-Item Inventory System with A Constraint for A Single Period.

by

Park, Hwa Jin
Major, Republic Of Korea Army
B.A., Korea Military Academy, 1981

Submitted in partial fulfillment of the
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ABSTRACT

This thesis addresses the problem of determining the optimal number of spares in a multi-item inventory system with a procurement budget constraint for a specific period. In war time, an expected shortages of spares is an extremely critical factor in the ROK Army's ability to fight. For the initial stage of war, the ROK Army combat units have to reserve the "loading package". The purpose of this thesis is to select the loading package to minimize the expected number of shortages over all the items included subject to a budget constraint. A Marginal Analysis and Dynamic Programming (DP) are considered. Though Marginal Analysis is not strictly optimal it is very nearly so and much more efficient than DP. Consequently, the Marginal Analysis Approach appears preferable in the military area because it is very simple, flexible, and easy to program.

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I. INTRODUCTION

Traditionally, in managers' efforts to reduce costs and improve profits they concentrated their efforts on the manufacturing part of the enterprise. Production techniques were constantly improved. Another important area of cost savings is in the non-manufacturing operations, and many business managers turn their attention to these costs nowadays. Those operations will include functions such as procurement, transportation, warehousing, inventory control and materials handling, all of which are considered integral parts of logistics.

In today's world, while all logistics systems are becoming more and more sophisticated, the control and maintenance of inventories of these systems is a problem common to all enterprises and military services. In commercial concerns both private and public, an effective inventory control results in decreased costs and increased consumer satisfaction. Especially in the military, the proper management of inventories contributes to increased availability of materials, and results in higher combat readiness as well as decreased inventory investment costs.

For each component of each weapon system, two fundamental questions must be answered:

- *When to replenish the inventory*
- *How many to buy for the replenishment*

In order to answer these questions, many inventory models have been developed. Most of the models developed in the past solve a variety of cost minimizing problems considering expected values of steady state variable costs associated with ordering cost, holding cost, and shortage cost. Such models may be appropriate for the commercial sector, but are not always proper in the military area. In the commercial sector, the purpose of inventory models is to maximize profit or minimize annual costs. In the military sector, the purpose is to achieve maximal readiness. Examples of non-cost oriented purposes used in the military are maximizing availability or fill rate, or minimizing the expected number of backorders, or minimizing a stockout with constraints such as budget, weight or volume, etc.

Generally speaking, many real-world inventory problems are so complicated, one cannot represent the real situation accurately or easily. Simplifications and approximations are used when constructing a mathematical model of any real world system. Without such simplifications the models would become unmanageable. Therefore, we will resort to such simplifications in this thesis.

The purpose of this thesis is to minimize the expected shortages in a multi-item inventory system with a budget constraint for a specific period--the initial stages of war -- and to recommend the marginal analysis method to select the "loading package" for war time in the Republic Of Korea (ROK) Army.

Chapter II briefly describes the history of the military logistics and the background of the inventory system of the ROK Army.

Chapter III explores the nature of inventory systems regarding the inventory problems and various inventory costs which are required in an inventory model.

Chapter IV introduces the marginal analysis theory, which is frequently used in economics, as it applies to inventory management. Next we will show procedures designed to apply the marginal analysis. Then we illustrate and compare the Marginal Analysis and Dynamic Programming for minimizing the expected shortages with a budget constraint.

Chapter V presents conclusions. It will emphasize the marginal analysis for minimizing shortages in multi-items inventory systems for the initial stages of wartime in the ROK Army.

II. AN OVERVIEW OF ROKA'S INVENTORY SYSTEM

A. HISTORY

Early on the morning of June 25th, 1950, North Korean forces launched on invasion of the ROK, which surprised all of the free world. The ROK Army was manning the 38th Parallel and had only five regiments in place. All had been reduced in strength because of long weekend passes that had been granted.

The ROK Army was no match for the North Korean forces. The heaviest weapons the ROK Army possessed were twenty-seven armored cars and eighty-nine 105 mm howitzers which were in good working condition but with short range. Fifteen percent of the ROK Army's weapons were useless and thirty-five percent of their vehicles were out of commission. Their ammunition was sufficient for a few days, including small arms stocks. The North Korean forces took three and a half days to seize Seoul. This initial success convinced the enemy that they could overrun the entire peninsula in two weeks.

ON June 30, the President of United States, Harry Truman announced that the United States Army forces were to be committed to ground combat in Korea. Thus, two United States divisions were ordered to Korea. These divisions were ill-equipped, had little ammunition, a supply base that had critical shortages and did not have a full complement of men. Later, sixteen United Nations countries forces were committed to combat in the defense of South Korea [Ref. 1 : p.615].

The principal source of supplies for United Nations forces in Korea was the United States. An important part of this United States' logistics support was by an extensive program of rebuilding World War II equipment, collected in Japan. A third major source of supply, important in filling gaps between demand and supply, was procurement from local merchants and manufacturers in Japan and Korea.

As the Korean conflict settled into an apparently endless stalemate, an alternative to the indefinite commitment of American forces was to build the ROK Army up so that it could relieve American divisions in the defense of the ROK and make a major contribution toward breaking the stalemate. With peace talks giving no promise of an early armistice, pressure toward that end began to increase on both sides of the Pacific in the spring of 1952. It was to become a theme for much of the discussion in the presidential campaign of that year.

General Clark, in response to a request from the Chief of Staff, had submitted a plan to expand the ROK Army to twenty divisions within a period of eighteen months. However, he did so with grave misgivings about the over-all logistical implications of such a build-up within such a short period of time. [Ref. 1 : p.638]

Even if the suggested expansion of ROK forces was successful in permitting the withdrawal of American forces, it was to be questioned whether such a course would have been wise from the point of view of international politics. What had begun as a common United Nations effort would then be completely lost, and the Korean War would be reduced to a civil war. At the same time, it was doubtful whether the South Koreans themselves would have been enthusiastic about the complete withdrawal of American combat divisions. Perhaps a more serious concern for the United States was the loss of material that would be involved. Weapons and equipment then in the hands of American and other United Nations forces in Korea could still be considered a material reserve for use elsewhere in the Far East, or in other parts of the world if necessary. But if they were handed over to the ROK they could not be counted on for use anywhere else in the world.

A further fact that could not be ignored was the danger that some material, if given to the ROK Army, might fall into the hands of the Communists. Doubtless the Chinese Communists would have the capability of overrunning even a 20-division South Korean Army if there were no other forces to deter them, and as had happened in the case of some of the Chinese Nationalist forces, equipment delivered for the defense of Korea might actually be used against it.

Moreover, there was no getting away from the fact that such a large-scale turnover of equipment to the ROK could not help but delay the fulfillment of the North Atlantic Treaty Organization programs by another year or more. It would also further delay the arming of Japanese security forces, would make necessary the continuation of the 50 percent ceiling on critical items for training units in the United States, and would continue the gamble of getting along without reinstituting the material reserves in the United States. Early in February 1953 President Eisenhower authorized General Clark to add two divisions to the ROK Army immediately to make a force of fourteen divisions plus six separate regiments with an over-all ceiling (including marines) of 507,880. A few days later the Joint Chiefs of Staff recommended to the secretary of defense that the program for expanding the ROK Army to twenty divisions be implemented immediately. [Ref. 1 : pp.637-638]

Through the Korean War (1950-1953) and participation in the Vietnam War (1968-1973), ROK could get the experience logistically to be able to wage modern war. Unfortunately, ROK obtained less experience in the logistics field because we waged the Korean War under support of the United States. and other United Nations countries, and also participated in the Vietnam War under the support of the United States. Little experience in logistics in both wars has resulted in neglecting the importance of logistics by the ROK Army.

Considerable effort and study have been devoted toward the development of credible strategic and tactical doctrine and training systems. Various strategies are continually under review to find better methods of projecting military capability under coordination with the United States. Through this process, one central question has usually been avoided and delayed: What about military logistics ? Logistics has been relegated to a category of secondary importance. The idea that "logistics supports the weapon system" or in fact that logistics provides "support" still prevails.

Moreover, when the ROK rebuilt its Armed Forces following the Korean War, it emphasized combat units rather than support units. While this has little effect during the opening days of war, it affects the tide of battle as the staying power of South Korean forces is weakened by thin logistics support.

A forward defense, in which the defender cannot afford to yield a great deal of territory, demands heavy artillery and mortar bombardment and close air support. Moreover, a heavy barrage of indirect fire uses up a lot of ammunition during the first days of a conflict, and the ROK conventional ammunitions in place may be inadequate. Also, the logistics network for moving supplies from the port of Pusan or airfields in Seoul to the combat area is very thin, and since they would come primarily from the United States, would require weeks of transit time.

The worst is as follows. The United States military experience showed that the Army would begin running out of key ammunitions and other items in the initial stage of a war in South Korea and could be forced to accept a stalemate because of shortages of critical supplies. Therefore, adequate stocks of conventional ammunition in the theater would go a long way to enhancing South Korea's defense capability. To increase these stocks, the United States could assist the South Korean Army in improving its ammunitions industry. The ROK began to build the defense industry in 1975.

United States security assistance to the ROK has played an essential role in strengthening the military capabilities of the ROK since 1953. In the past, the ROK Army's main focus of defense management was to obtain as many resources as possible from the United States. Therefore, most decision makers (or managers) in the ROK Army headquarters paid more attention to the effective and efficient contribution of the resources obtained from the United States and less attention to the more effective and efficient procurement, inventory, and management methods. They didn't feel the need for developing an advanced management system and training specialists to enhance defense productivity.

While strategies and tactics are quickly adopted to changing situations and objectives, the logistics system always needs a large and complex process requiring complete and careful management to make sufficient changes in objectives or results. Today, knowledgeable logistics leadership, guidance, and policy are needed to create a single logistics operation focused on attaining necessary military capability in the ROK Forces.

In July 1983, the ROK President Chun, Too Whan issued an executive order requiring the Korean armed forces to reduce the operating expenditure in the nation's defense budget. This was done In order to provide funding for an increase in capital investment [Ref. 2 : p. 12]. The objective of this order was to take initial steps toward improving defense resource management.

Table 1. COMPARISON OF MILITARY POWER INDICES BETWEEN SOUTH AND NORTH KOREA: IN PERCENTS (NORTH KOREA = 100)

YEAR	'76	'77	'78	'79	'80	'81	'82	'83	'84	'85
ARMY	58.4	58.5	58.5	58.7	59.1	59.3	59.6	61.3	63.8	60.9
NAVY	42.5	47.1	47.2	45.2	49.7	49.3	53.1	59.5	55.8	59.4
A.F.	41.8	38.9	43.3	43.2	42.0	43.8	51.1	51.9	52.7	59.9
TOTAL	51.9	52.3	53.2	52.9	53.8	54.2	56.6	59.1	60.2	60.5

While the ROK economy has made excellent progress, defense spending is reaching about 6% of GNP or approximately one third of the government budget. This figure is considered high by free-world standards. Despite this high level of spending, the imbalance of military power between the two Koreas remains in favor of the North as shown Table 1 [Ref. 2 : p. 20].

As a means of obtaining greater defense capabilities to balance military power between South and North, the government of ROK has taken steps to obtain greater efficiency in its use of defense funding. Promoting the efficiency of defense operating expenditures will be the best way to accomplish the objective of self-defense.

1. BACKGROUND

For want of a nail, the shoe was lost,
For want of a shoe, the horse was lost,
For want of a horse, the rider was lost,
For want of a rider, the battle was lost,
For want of a battle, the kingdom was lost,
And all for the want of a horse-shoe-nail.
-Margaret de Angelis, 'Mother Goose'-

Unfortunately, to this day the problem remains - if anything it is even worse than before. Many a critically important electronic "kingdom," such as a radar, a computer, or a missile system, has been rendered useless during critical periods of need because of a shortage of an electronic "nail," say one dollar resistor. This thesis attempts to prevent the loss of "kingdoms" by showing how to insure that enough "nails" are on hand - without paying a king's ransom.

It is increasingly apparent that one of the key problems facing large military and industrial organizations is to prevent the paralysis of complex systems in the field resulting from the shortage of some essential component. The problem stems from the fact that component failure rate is essentially random, so that one cannot predict with certainty the number of replacements required for each type of essential component. If the system has been in operation for some time, then experience may have been accumulated as to the level of demand for replacements; in this case, rational procedures exist for determining sensible spare part kits. If the number and variety of essential components is great, and has constraints, say money available to purchase, or loading capacity on hand to transport spare parts is limited, then the problem becomes quite difficult.

The primary purpose of logistics in the military is to deliver adequate potential or actual power or shock to the critical places at the critical times for achievement of tactical and strategic objectives. It is, in short, "to get there first with the most." All else is subordinate to this primary purpose.

Rapid changes in weapons and equipment in the more recent periods make it necessary to modify this statement somewhat. Although "to get there first with most" probably is a fair expression of the primary concern of Army logistics over the years, it is not enough to be first with inferior weapons and equipment. Although quantity frequently has made up for lack of quality, and sometimes vice versa, the ideal has been to have both, so that now this primary aim of logistics might better be stated, "to get there first with the most and the best".

During the Korean War, no single item of supply received more attention in the support of operations in Korea than ammunition. At that time, the ROK Army's ammunition supply system was based on a "continuous refill system" according to which each unit carried a prescribed "basic load of ammunition" set by the theater commander, which was replenished as used. The basic load was supposed to be an amount that the unit could carry with its own transportation, and it was the responsibility of each unit commander to maintain it. Ordinarily the ammunition supply point filled orders consisting only of a statement on a transportation order that certain ammunition was necessary to replenish the basic load. Successive commanders, from theater commander down to lower units, controlled the consumption of ammunition by prescribing "available supply rates of ammunition." These rates, expressed in rounds per weapon per day in the tactical units, represented the consumption that could be sustained with available supplies. In addition, commanders made up estimates of the number of rounds per weapon per day needed to sustain operations of a designated force without restriction for a specific period. This estimate, referred to as the "required supply rate of ammunition" was the basis for operations planning. This ammunition system is still used in the ROK Army today.

In the 1950s and 1960s, arms transfers from the United states were made under the Military Assistance Program; during the subsequent period of the ROK Armed Forces Modernization Program (1971-1975), the military capabilities of the ROK were greatly enhanced under such the United States Military Aid Program and the Military Assistance Service Fund.

Since the Mid-1970s, the United States security assistance policy toward ROK has undergone a tremendous change. Assumption of increased responsibility for its own defense made the ROK Department Of Defense realize the need for a better and more efficient allocation of defense resources. Toward this end, the ROK Army studied and improved the U.S.'s Planning, Programming, and Budgeting System (PPBS). Finally, the Planning, Programming, Budgeting, Executing, and Evaluation System (PPBEES) unique to the ROK military needs was developed [Ref. 3 : p. 86].

The concept of PPBEES is to design a bridge between the planning and programming phases and to feed back the results of performance evaluations for use in subsequent phases of planning, programming, and budgeting.

Under PPBEES, increments to programs were considered in the programming phase and these increments were then translated into line item entries for the traditional budget submission [Ref. 4 : p. 91]. This PPBEES system aims to develop a sound Defense Resource System by adding the execution and evaluation phases to the previous budget system.

According to the system, a commander of a unit has to manage his troop within the allocated budget regarding all activities. so, it is natural that he should strive to use effective and efficient management techniques.

III. THE NATURE OF INVENTORY SYSTEMS

A. PURPOSES OF INVENTORY

The control and maintenance of inventories of physical goods is a problem common to all enterprises in any sector of a given economy and all military sectors. For example, inventories must be maintained in agriculture, industry, retail establishments, and the military. In the United States, it has been estimated that over the decade of 1970-1979 the average annual investment in business inventories was $305 billion, or 18 percent of the gross national product. The cost of carrying these inventories in the business sector has been estimated at between 30 and 40 percent of inventory value before taxes. This was an annual cost to the nation of between $128 and $170 billion in 1979 [Ref. 5 p.356].

Among individual firms, inventory carrying costs can represent 10 to 40 percent of total logistics cost, depending on whether the firm is a manufacturing, merchandising, consumer, or an individual goods-oriented company.

We may look on inventory as a necessary evil even though it is listed in the accounts of the firm as an asset. Like all assets, it ties up capital which can be put to other uses. The very existence of an inventory creates costs.

On the other hand, if we were to buy the smallest available quantity of each item we use in our personal lives or business, we would be spending all our time buying things and still would have insufficient time to buy everything we needed and no time to fulfill the purposes of our lives or business. Therefore, one reason for having inventories is to save time by avoiding a prohibitively large procurement work load.

Often inventories are carried to balance incoming materials against production schedules or sales. It is rarely possible to maintain the same rate of input and output. In order to keep a production line operating continuously, or to keep sufficient stocks to meet sales demands with a minimum of lost sales due to stockouts, a certain amount of inventory must be maintained. In a sense, inventories uncouple the supplier and customer. The inventory acts as a buffer between a supplier who generally supplies materials in large amounts (relative to one's daily or weekly demand or consumption) and a user (consumer or production line) who generally buys the output or uses the material in smaller or more variable quantities.

Furthermore, the price of goods is a prime consideration in establishing an inventory. Large quantities are sometimes available at discount prices. Quantity discounts sometimes provide an additional reason for holding inventories for future use rather than buying only to satisfy immediate needs. It is because that the additional costs of having, housing, and holding larger stocks do not offset the price advantage (and potential work-load reduction).

B. TYPES OF INVENTORY

Inventory may consist of supplies, raw materials, in-process goods, and finished goods. Supplies are inventory items consumed in the normal functioning of an organization that are not a part of the final product. Typical supplies are pencils, paper, light bulbs, typewriter ribbons, and facility maintenance items. (Factory supplies are called MRO, for maintenance, repair, and operating supplies.) Raw materials are items purchased from suppliers to be used as inputs into the production process. They will be modified or transformed into finished goods. For instance, typical raw materials for a furniture manufacturer are lumber, stain, glue, screws, varnish, nails, paint, and so forth. In-process goods are partially completed final products that are still in the production process. They represent both accumulation of partially completed work and the queue of material awaiting further processing. Finished goods are the final product, available for sale, distribution, or storage.

Table 2. TYPES OF INVENTORY

Input Source	Inventory Type	Output Destination
Suppliers	Supplies	Admin., Maintenance
Suppliers	Raw materials	Production
Production stages	In-process goods	Next production stage
Suppliers, Production	Finished goods	Storage, Customer

The assignment of inventory to any of these categories is dependent on the entity under study. This is because the finished product of one entity may be the raw material of another. For example, a refrigerator manufacturer considers copper tubing as a raw material, but the firm that produces the tubing considers it as a finished good. The customer for finished goods inventory may be the ultimate consumer, a retail organization, a wholesale distributor, or another manufacturer. Table 2 [Ref. 6: p.5] indicates the types of inventory.

C. THE INVENTORY COSTS

The costs incurred in operating an inventory system play a major role in determining what the operating policy should be. The costs which influence the operating policy are clearly only those costs which vary as the operating policy is changed. Costs that are independent of the operating policy used, need not be included in any analysis where costs are used as an aid in determining an operating policy [Ref. 7: p.10].

The objective of inventory management is to have the appropriate amounts of raw materials, supplies, and finished goods in the right place, at the right time, and at low cost. Inventory costs are associated with the operation of an inventory system and result from action or lack of action on the part of management in establishing the system. They are the basic economic parameters to any inventory decision model, and the more relevant ones to most systems are itemized as follows: [Ref. 6: p.7].

- Procurement cost
- Order set up cost
- Holding cost
- Stockout cost

Note that for a particular inventory item, only those cost elements that are incremental (out of pocket) are pertinent in the analysis.

1. The Procurement Costs

Procurement (or purchase) costs associated with the acquisition of goods for the replenishment of inventories are often a significant economic force that determines the reorder quantities. When a stock replenishment order is placed, a number of costs are incurred, that are related to the processing, transmitting, handling, and purchase of the order. More specifically, procurement costs include the price, or manufacturing cost, of the product for various order sizes; the cost of processing an order through the accounting and purchasing department; the cost of transmitting the order to the supplier, usually by mail or by electronic means; the cost of transporting the order when transportation charges are not included in the price of purchased goods; and the cost of any materials handling or processing of the order at the receiving dock. When the firm is self-supplied, as in the case of a factory replenishing its own finished goods inventories, procurement costs are altered to reflect production setup costs. Transportation costs may not be relevant if a delivered pricing policy is in effect.

Some of these procurement costs are fixed per order and do not vary with order size. Others, such as transportation, manufacturing, and materials-handling costs, vary to a degree with order size. Each case requires slightly different analytical treatment.

2. The Order Costs

The order or setup cost originates from the expense of issuing a purchase order to an outside supplier or from internal production setup costs. This cost is usually assumed to vary directly with the number of orders or setups placed and not at all with the size of order. The order cost includes such items as making requisitions, analyzing vendors, writing purchase orders, receiving materials, inspecting materials, following up orders, and doing the paperwork necessary to complete the transaction. The setup cost comprises the costs of changing over the production process to produce the ordered item. It usually includes preparing the shop order, scheduling the work, preproduction setup, expediting, and quality acceptance.

3. The Stockout Costs

The stockout cost (depletion cost) is the economic consequence of an external or an internal shortage. An external shortage occurs when a customer's order is not filled; an internal shortage occurs when an order of group or department within the organization is not filled. External shortages can incur backorder costs, present profit loss (potential sale), and future profit loss (goodwill erosion). Internal shortages can result in lost production (idle man and machines) and a delay in a completion date. The extent of the cost depends on the reaction of the customer to the out-of-stock condition. If demand occurs for an item out of stock, the economic loss depends on whether the shortage is backordered, satisfied by substitution of another item, or canceled. In the one situation, the sale is not lost but only delayed a few days in shipment. Typically a company would expedite an emergency backorder for the item and assume any extra costs charged for the special service (e.g., expediting costs, handling costs, and frequently premium shipping and packaging costs). In another situation, the sale is lost. The actual cost is less identifiable in this case but ranges from the apparent profit loss on the sale to loss of goodwill, which can be hard to specify. It can be seen that the stockout cost can vary considerably from item to item, depending on customer response or internal practice. It can be extremely high if the missing item forces a production line to shut down or causes a customer to go elsewhere in the future. The quantitification of these costs has long been a difficult and unsatisfactorily resolved issue.

The central objective of inventory management (though not the sole objective) is usually the minimization of costs. Only those costs which change as the level of inventory changes should be considered in any analysis. For example, amounts expended on heating, lighting, and security services for a warehouse should be disregarded if they do not change as stock levels vary.

4. The Holding Costs

The holding cost, synonymous with carrying cost, subsumes the costs associated with investing in inventory and maintaining the physical investment in storage. It incorporates such items as capital costs, taxes, insurance, handling, storage, shrinkage, obsolescence, and deterioration. Capital cost reflects lost earning power or opportunity cost. If the funds were invested elsewhere, a return on the investment would be expected. Capital cost is a charge that accounts for this unreceived return.

The inventories are usually treated as taxable property; so the more you have, the higher the taxes. Insurance coverage requirements are dependent on the amount to be replaced if property is destroyed. Insurance premiums vary with the size of the inventory investment. Obsolescence is the risk that an item will lose value because of shifts in styles or consumer preference. Shrinkage is the decrease in inventory quantities over time from loss or theft. Deterioration means a change in properties due to age or environmental degradation. Many items are age-controlled and must be sold or used before an expiration date (e.g., food items, photographic materials, and pharmaceuticals).

The usual simplifying assumption made in inventory management is that holding costs are proportional to the size of the inventory investment. On an annual basis, they most commonly range from 20% to 40% of the investment. In line with this assumption is the practice of establishing the holding cost of inventory items as a percentage of their dollar value [Ref. 6: p.14].

Table 3 [Ref. 8: p.5] indicates various the holding costs.

Table 3. CHARACTERISTICS OF THE HOLDING COST

Classification	Nature of Cost Involved
Interest	Capital to produce or purchase inventory items must be secured by (1) borrowing from a bank (2) reducing other planned uses of the money (such as buying new machinery or equipment, increasing sales and promotional efforts). While some firms charge an accounting figure of from 15 to 20 percent per year (to themselves) on capital tied up in interest, a rate of from 8 to 12 percent is probably more realistic.
Obsolescence, Deterioration, Spoilage	Few items improve with age. In an age of rapidly developing technologies, there is always the danger of equipment or products becoming out of date. A new invention or a new synthetic material may, on occasion, cause overnight obsolescence. Then there is always the danger of deterioration, spoilage, and damage from various types of insects, from heat, cold, or other causes.
Storage and Materials handling	Storage is expensive, regardless of whether it consumes space in one's own building (which could be used for more profitable alternatives) or is rented. Moreover, storage is rarely a one-time proposition. As new materials are received, shifting and changing of storage areas and locations may become necessary.
Insurance	Fire, windstorm, floods, theft, explosions, and other hazards require insurance coverage.
Taxes	Taxes vary with localities. They involve not only the items in storage , but also the building used for this purpose.
Record systems	Materials in storage must be accounted for. Periodic checks are required, not only for quantity but also for possible deterioration of merchandise. We must know where each item is stored, from original receipt through various moves within the warehouse, and we must be able to retrieve the item when needed. Quantities must be known. Valuations may need periodic revising. Although much of this work may be computerized, there is nevertheless the need for hand-prepared reports on individual transactions (movement in or out, transshipment within or between warehouses or other storage areas).
Total	Over-all storage costs may represent from 10 to 20 percent or more of the value of items inventoried.

IV. MARGINAL ANALYSIS APPROACH

A. CONCEPT OF A MARGINAL ANALYSIS

1. Conditions for the Use of the Marginal Analysis

In the previous chapter, we looked into the various costs that affect the inventory models. In Appendix A we describe three typical inventory models, a deterministic model, a stochastic model, and a single period model, which are derived from the situation that has an objective function for minimizing the average annual variable inventory costs such as the holding cost, the ordering cost, and the shortage cost (Refer to Appendix A).

We now turn our attention to our main problem, namely a multi-item inventory system with a budget constraint. The essential problems of control in a line item inventory control system with the multi-items are:

- How many resources to commit at a given point in time.
- How should these resources be allocated among the diverse opportunities afforded by the various items to achieve system objectives.

In a typical continuous review inventory system, we can determine the optimal order quantity (Q_0) and reorder point (R) for a given time by minimizing the average annual variable costs. However, in applying this model to the real world inventory systems which consist of multiple line items with a budget constraint, it is frequently the case that resulting minimum cost solutions are not feasible because of a budget limitation or some other constraint. Thus, in the constrained multi-item inventory system, the typical continuous-review policy is sometimes inappropriate.

In this chapter, we consider the situation that the ROK Army currently meets, which may present unexpected difficulties to the ROK Army logistics in the initial stages of the future war.

Since the Korean War, the ROK has procured most weapon systems from the United States. The ROK has developed and mass-produced some important weapon systems proper to Korean terrain except for high-tech weapon systems since 1972 [Ref. 9 : pp. 19-30].

When the ROK purchases a weapon system from abroad, they feel that it is important to support the weapon system with certainty because they do not produce the items needed to support it. While the ROK mass-produces some weapon systems, they may have to face some difficulties in making technical manuals (TM), providing various data, and so on. If the ROK procures a weapon system from the United States, the software described above is naturally included. Nowadays the ROK must provide all software to support and maintain the self-produced weapon system.

We now consider a particular situation--selecting the "loading package".[1] The ROK Army combat units currently reserve the spare parts for weapon systems with multi-item used with the predetermined standards[2] for the initial stage of war. In the case of imported weapon system, the amount of spare parts included in the loading package can be determined by its TM. However, in the self-production case, the ROK Army has to determine them by results of the test and evaluation of the weapon system in the pre-equipment stage.

In war time, the expected shortage of spares is an extremely critical factor in the ROK Army's ability to fight in the view of logistics. Therefore they need to develop the proper model for the following circumstances.

- It is possible to make all adjustments in the composition of the "loading package" prior to the use period (i.e., the first thirty days of war-time.) in which the inventory is to be used.
- It is impossible to re-supply items during the period of use.
- There are no substitutions allowed.
- The demand for the items in the inventory is independent of the quantities stocked.
- There is a budget constraint which limits the size of inventory.

The objective is to select the kinds and quantities of spare parts to include in a package of given total budget size so that the number of shortages of parts would be minimized. In this case, the descriptions of the situation fit well the assumptions of the static marginal analysis [Ref. 10 : pp. 156-188]. Indeed, it is possible to obtain reasonable estimates of marginal cost and marginal product. Therefore the marginal analysis theory can be applied to this case. What is the main idea of marginal analysis theory ? We consider it briefly.

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[1] The expected amount of items which are needed to support the weapon system without replenishment for a period of thirty days in war-time.

[2] These data are often cited from the U.S. technical manual which comes with the weapon systems

2. A Marginal Analysis Theory

Suppose that Q is the output rate and that the production function is $Q = f(x_1, x_2)$, where x_1 and x_2 are the amounts of the two inputs used. If we want to find the values of x_1 and x_2 that maximize Q for the given cost, C_0, we set up the Lagrangian function:

$$L = f(x_1, x_2) - \lambda(P_1 x_1 + P_2 x_2 - C_0)$$

where

P_1 = the price of the first input x_1

P_2 = the price of the second input x_2

λ = the Lagrange multiplier

The first order conditions are

$$\frac{\partial L}{\partial x_1} = \frac{\partial f}{\partial x_1} - \lambda P_1 = 0$$

$$\frac{\partial L}{\partial x_1} = \frac{\partial f}{\partial x_2} - \lambda P_2 = 0$$

It follows that

$$\frac{\partial f / \partial x_1}{P_1} = \frac{\partial f / \partial x_2}{P_2} = \lambda \qquad (1)$$

If we want to minimize the cost for the given input, Q_0, we set up the Lagrangian function:

$$P_1 x_1 + P_2 x_2 - M\,[f(x_1, x_2) - Q_0]$$

where

M = the Lagrange multiplier

The first order conditions are:

$$P_1 - \frac{M\partial f}{\partial x_2} = 0$$

$$P_2 - \frac{M\partial f}{\partial x_2} = 0$$

and therefore

$$\frac{\partial f/\partial x_1}{P_1} = \frac{\partial f/\partial x_2}{P_2} = \frac{1}{M} \qquad (2)$$

If we set $\lambda = \frac{1}{M}$ then Formula (2) is the same as Formula (1). If the output level is the same, the values of x_1 and x_2 that are chosen must be the same regardless of whether the output is maximized for the given cost or the cost is minimized for the given output.

This means that the marginal product of a dollar's worth of any one input is equal to the marginal product of a dollar's worth of any other input used. It is expressed mathematically as

$$\frac{\mathrm{MP}_1}{P_1} = \frac{\mathrm{MP}_2}{P_2} = \ldots = \frac{\mathrm{MP}_n}{P_n}$$

where

$\mathrm{MP}_i =$ the marginal product of *i*th input

$P_i =$ the price of *i*th input

This theory has been largely limited to economic theory and is rarely if ever applied to actual decision situations because of the inability of a simple static model to describe adequately real decision problems and because of the difficulty of measuring marginal cost and marginal product. However, as we mentioned earlier, the assumptions of static marginal analysis (such as demand independent of the decision, no substitution in use possibilities, one scarce resource, and no problem of discontinuous variables) also seem to be fairly well approximated in this special case as described in the previous section.

In general, when the objective function is to minimize the expected number of shortages for a specific time period. According to Howard [Ref. 11 : pp. 3-6]. The problem can be stated mathematically as

$$\text{Minimize} \quad Z = \sum_{i=1}^{n} \pi_i \sum_{X=S_i}^{\infty} (X - S_i)\, P_i(X)$$

subject to

$$\sum_{i=1}^{n} C_i S_i \le B$$

with

$$S_i \ge 0 \qquad\qquad i = 1, 2, \ldots, n$$

where

n = the number of different items

$P_i(X)$ = the probabilty that X units of item i will be demanded

π_i = the weight (essentiality) of a stockout for item i

C_i = the unit cost for the ith item

X = the demand for a item

S_i = the number of stocks of item i

B = the budget allocated to purchase inventory.

If we treat each item as equally important to a weapon system, then all π_i will be 1. One of the problems associated with providing an initial inventory is the lack of knowledge concerning the underlying demand generation probability distribution function. This lack of knowledge usually leads to the use of an assumed distribution or to an inventory based on expected values attained from the test and evaluation of sample systems in the pre-equipment stage in the ROK Army. The next section will deal with this problem.

Demand data for a sample of 200 items out of a total of about 1800 items was analyzed (Refer to Appendix B). Under the assumption that all items follow the same type of distribution, the Poisson distribution was found to provide an excellent fit of the actual data.

The marginal analysis theory merely states that an efficient mix of productive inputs is that mix for which the ratio of marginal product to marginal cost is the same for each input. In other words, the composition of productive inputs should be arranged in such a way that the additional value (marginal product) obtained from the last dollar's worth (marginal cost) of each input should be equal. Thus, the composition of the "loading package" should be such that inclusion of an additional unit of an item is only dependent on the decrease in expected marginal protection against stockout per budget dollar consumed. This can be expressed mathematically as

$$\frac{\mathrm{MP}}{C_i} = \frac{1 - P(X \leq n-1)}{C_i} \qquad (n \geq 0, 1, \ldots..)$$

where

$1 - P(X \leq n-1)$ = the marginal protection of nth stock of a item

X = the demand

C_i = the cost of item i

The marginal analysis procedure progressively assigns a unit to the inventory of that item which yields the greatest reduction in expected stockout probability per unit increase in budget usage. The following procedure is used for application of the marginal analysis.

3. A Procedure of Applying Marginal Analysis

Step 1

First of all, we have to find out each demand and cost of items for a specific period (i.e., 30 days in this thesis). We can figure them out from the data collected through test or operations during the past mission duration.

Step 2

Second, we consider the type of demand generation probability distributions. In this thesis, we assume that the probability distribution fits the Poisson distribution because the 30 days-demands of the items to be treated is less than 15.

Step 3

Third, we calculate the "marginal protection", which is computed from the probability of the possible demands of each item. This measures the additional product or value provided by each unit.

Step 4

Fourth, we have to compute the "marginal protection per dollar". This can be calculated simply by "marginal protection" divided by its unit cost.

Step 5

Fifth, we arrange the "marginal protection per dollar" in descending value. So, the greatest value becomes the first item.

Step 6

Finally, we select the kind and number of a item within the budget, which means that the cumulative total costs do not exceed the budget limit.

Now, we deal with a particular problem as an illustration of the marginal analysis application.

B. AN ILLUSTRATION

1. Problem Definition

A few years ago, the Korean Army asked for a specific Armored Personnel Carrier (APC) which is suitable to Korean terrain characteristics and satisfies the mission profile well. At last, the XK-1 APC is produced and will be used with two mechanized infantry divisions in the Korea Army. A battalion in the division consists of 58 XK-1 APC's. The battalion has to select the "loading package" of the APC for the initial stage of war. According to the current operational plan, if war breaks out, the battalion could not receive a re-supply of any repairable items before the end of the first month (30 days). Therefore, the battalion has to reserve some repairable items within its capability to repair them when they will be needed to repair. What kinds of repairable items should be included in the package? How many items should be selected?

A loading package of APC's spares is intended to supply a certain number of APC, say fifty-eight, for a specific period, say thirty days. It must also be possible within a few hours after DEFCON-II[3] to deploy in specified points of operation far from the origin and to service combat vehicles with the loading package as the only substantial source of spare parts. The costs of such a spares package for fifty-eight APC's, which contains hundreds of different kinds of items, should not exceed the budget limit. It is difficult by unaided judgement to determine the appropriate quantity of each item to be included.

One problem is that each different kind of item incurs a different cost when it is included in the package. Another problem is that there is statistical uncertainty as to the exact quantity of each item that will be required. We assume that these characteristics of circumstance are as the situation described earlier in this chapter.

Now, for the purpose of illustration it may be useful to present first a brief example of how statistical uncertainty affects the problem; Second, to describe how this statistical uncertainty can be used to measure marginal product, and finally, how this adds to the cost of including an item which can be fitted into an analysis which equals the ratio of marginal product to marginal cost for every item.

Table 4 and Figure 1 show the distribution of observed demands for XK-1 APC spares generated by one battalion (58 APC's) during a one year-test period. Only items which had demands of 30 or less are shown since it is these parts with relatively low demand rates that are most seriously affected by statistical uncertainty. As can be seen from the chart, the tendency is for the items to pile up at extremely low demands. During a 696 APC-month period, 411 different items had no demand. About 300 different items were demanded only once and 147 different items were demanded twice. Six items were demanded 30 times, and so on.

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[3] This is the second class of the defense condition

Table 4. **NUMBER OF DEMANDS VS. NUMBER OF ITEMS** (Approximately 1800 installed main items.) Source : Analysis of Logistics for XK-1 APC (pp. 47-48)

# Demands	# Items	# Demands	# Items	# Demands	#Items
0	411	11	23	22	11
1	301	12	22	23	12
2	147	13	23	24	11
3	42	14	20	25	14
4	51	15	13	26	12
5	46	16	19	27	6
6	21	17	14	28	12
7	35	18	10	29	6
8	25	19	13	30	6
9	30	20	15	.	.
10	24	21	7	.	.

The low demand items (items with demands of 30 or less during the 696 APC's-month period) represent about 80 percent of the possible candidates (1402 items of approximate 1800 installed main items) for the loading package. Because of their low demand rates they only account for about 22 percent of the quantity of material consumed, but the 22 percent is extremely important as can be seen from examining data on APC out of mission for parts (shortages). [Ref. 12 : pp. 45-49.]. Such data were investigated for a period immediately following the test duration which the sample in Table 1 was collected. The report [4] of the test indicates the fact that the factors of most mission-down APC's is out of these low demanded parts, in other words, it was found that most causes for a mission-down condition was the stockout of these items during the previous 696 APC-months of operation.

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[4] See Ref.11 pp. 32-49.

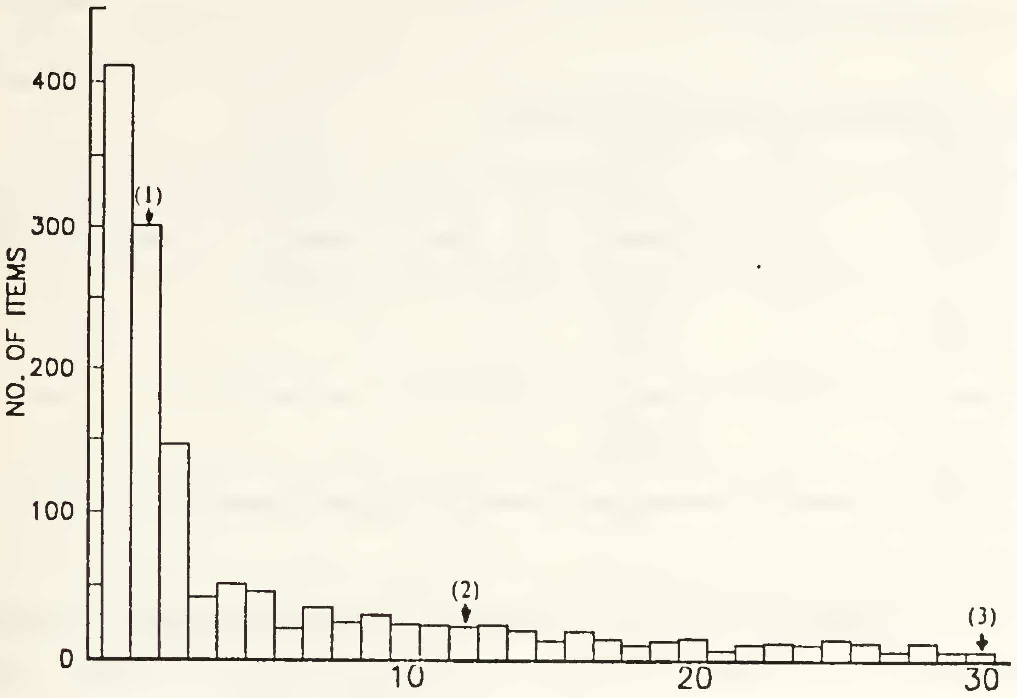

Figure 1. The Distribution of Observed Demand Rates for XK-1 Spares

The reason that these low demanded items cause so much trouble is due to the effect of statistical uncertainty combined with the fact there are so many of them. How does this occur ? To illustrate "how" and to show how the problem can be approached by marginal analysis, it will be convenient to examine a group of three typical parts from the distribution of demand rates shown in Figure 1. These three groups of parts are indicated by the number (1), (2), (3).

2. Statistical Considerations

The 301 items indicated by (1) were each demanded only once during the 696 APC-months. This is the demand rates of one battalion of 58 APC's. So, the demand rates per month would be equivalent to $\frac{1}{12} = 0.0833$. While 0.0833 represents an extremely low demand rate, nearly 40 percent of APC spares have demand rates this low and lower. And, about 45 percent of the APC out of mission for spares are caused by this same group of parts with low demand rates. The 23 items in the 2nd group indicated by (2) were demanded 12 times during the period. This would be equivalent to a demand rate of one per month. The 6 items in the last group were demanded 30 times during the same period. This represents an average demand of 2.5 parts per month. Table 5 shows this in detail.

Table 5. DEMANDS PER EACH PERIOD

Period	Item Group (1)	Item Group (2)	Item Group (3)
1 Year	1	12	30
1 Month	0.0833	1.0	2.50

The right side of Table 5 shows the behavior of items of group (3) that have an average demand of 2.5 parts per month. Of course, an item with an average demand of 2.5 parts per month does not experience exactly 2.5 demands per month. Some months there is no demand, sometimes there is only one demand, sometimes two, sometimes three, and so on. Over a period of many months, the demand average would be 2.5, but in any given month the demand may be zero, one, two, three, or even more. These potential demands are shown in the last column in Table 6.

Demand (X)	Average Demand per Month (λt)		
	0.0833	1.0	2.50
0	0.9201	0.3679	0.0822
1	0.0766	0.3679	0.2052
2	0.0032	0.1839	0.2565
3	0.0001	0.0613	0.2138
4	0.0000	0.0153	0.1336
5	0	0.0031	0.0668
6	0	0.0005	0.0278
7	0	0.0001	0.0099
8	0	0.0000	0.0031
9	0	0	0.0009
10	0	0	0.0002
11	0	0	0.0000

Table 6. DISTRIBUTION OF PROBABILITIES OF X DEMANDS.

Values of $P(X = x|\lambda t) = \frac{(\lambda t)^x e^{-\lambda t}}{x!}$

(Values less than 0.00005 are shown as 0.0000.)

A Poisson process produces a discrete number of occurrences in a continuous interval. The interval may refer to any continuous measure of time, distance, areas, or the like. The random variable of a Poisson process is the number of occurrences (X) within the interval [Ref. 13 : p. 190]. In this case, the particular probabilities shown in Table 1 are computed from the theoretical Poisson distribution. This distribution is widely used in production quality control and inventory control to describe the probability of occurrences of failures or demands [Ref. 14].

Analysis of XK-1 APC data has indicated that a large proportion of the demand for the vehicle spares also follows a pattern similar to the Poisson distribution. The Poisson distribution is a particularly convenient distribution to use since its probabilities are completely determined by the average demand rate (λt). This can be expressed mathematically as

$$P(X = x|\lambda t) = \frac{(\lambda t)^x \, e^{-\lambda t}}{x!} \qquad \text{for} \qquad x = 0, 1, 2, \ldots\ldots$$

where

λ = the average demand per time

x = the possible number of demands

e = 2.71828.....

In Table 6, the columns show the probabilities of each level of demand. For example, in item group (1). 301 items, the probability of a zero demand during a given month is

$$P(X=0|0.0833) = \frac{(0.08330)^0 \, e^{-0.0833}}{0!} = 0.9201$$

This means that the theoretical probability of no failure would be about 92 percent, similarly, the odds are about 77 in 1000 that there will be a demand for one unit. about 3 in 1000 that two units will be required. There are only 77 chances out of 1000 that the demand for one of these item group (1) parts in a particular month would be equal to the average demand. There is more than 90 percent probability that the demand would be different. This statistical uncertainty helps explain some of the difficulties that would be encountered if the quantity stocked were based on an average 30 day-demand. Unfortunately it is crucial to the combat power in the initial stage of war time.

Therefore, if we stock each one in group (1), the expected surplus would be 277 items ($= 301 \times 0.9201$), only 24 items ($= 301 \times (1 - 0.9201)$) would be used.

The probabilities shown in Table 6 are for a single item. Each of the individual 301 items having an average demand rate of 0.0833 per month is subject to these same probabilities. Table 7 shows the expected supply results from all 301 kinds of items combined if the items are not stocked at all and if the average monthly demand (0.0833 per month) is stocked. In fact, because the number of stocks must be an integer, we take one.

Table 7. EXPECTED SUPPLY RESULTS FOR ITEM GROUP (1)

Policy	# Consumption	# Surplus	# Shortages
Stock 0	0	0	25
Stock 1	24	277	1

If none of these 301 different kinds of items are included in the package, then there can be no surplus of the items and of course, none of them will be used, but the expected number of shortages resulting from these items is 25 units, which is only the probability of one or more demand during a given month times the number of kinds of items (301 items). Mathematically,

$$E(s) = N \sum_{X=S+1}^{\infty} (X - S) \times P(X)$$

$$= N\sum_{X=1}^{3}(X - 0) \times P(X)$$

$$= 301 \times [(1 - 0) \times 0.0766 + (2 - 0) \times 0.0032 + (3 - 0) \times 0.0001]$$

$$= 25.0733 \quad \text{units}$$

where

$E(s) =$ the expected number of items,

$N =$ the number of items,

$X =$ the possible demands,

$S =$ the number of stocks to be included in the package,

$P(X) =$ the probability that X units will be demanded.

Though the expected demand is 25, this does not mean a shortage of one unit of each kind of 25 of 301 items because it may happen that the demand for a particular item will be two or more. In the long run, the average sum of all the shortages together will add up to approximately 25 units.

If one unit of each kind of item in group (1) is put into the loading package, the supply results to be expected are that there will be 277 surplus parts;

E(surplus) = (the number of stocks) × (probability of zero demand)

$$= 301 \times 0.9201$$

$$= 276.9501 \qquad \text{units}$$

and that 24 parts will be used:

E(consumed) = (the number of stocks) × (Σprobability of one or more demands)

$$= 301 \times 0.0799$$

$$= 24.0499 \qquad \text{units}$$

and that there will be one requirement which cannot be satisfied and results in supply shortages. These expected shortages result from the items whose demands are greater than one.

$$E(s) = N\sum_{X=2}^{\infty}(X-1) \times (PX)$$

$$= 301 \times [(2-1) \times 0.0032 + (3-1) \times 0.0001]$$

$$= 1.0234 \qquad \text{units}$$

Table 8 shows the expected supply results for item group (2).

Table 8. EXPECTED SUPPLY RESULTS FOR ITEM GROUP (2)

Policy	# Consumption	# Surplus	# Shortages
Stock 0	0	0	23
Stock 1	15	8	8

As mentioned in the previous Table 7, if one unit of these 301 items were put into the loading package, it would represent an average one year supply. The expected results from stocking one unit of each of these 301 items are that there would be 277 parts which are surplus, 24 parts consumed, and 1 supply failure. Because it is impossible to know which part will be demanded, it would be necessary to carry 301 parts all, 277 of which are not used, in order to supply 24 out of 25 demands (expected shortage when zero stocking). In spite of this large surplus (92 percent), one supply failure is expected to occur. At a low demand rate like this, the surplus problem becomes acute. Since the loading package is limited in size. This example indicates the effect of statistical uncertainty on proper selection of stock for the loading package.

3. Computation of Marginal Product and Marginal Cost

We now consider how to compute the marginal product and marginal cost. The limitation on size of stocks is what makes all economic decision problems important and interesting. If it were feasible to use infinite amounts of every productive input there would be no decision problems worth considering. The limitations or constraints on any productive activity can usually be expressed ultimately in terms of dollar cost. Sometimes this limitation may expressed in terms of some other resources such as weight, storage space, man-hour, or elapsed time.

In the case of the loading package problem, it is a budget constraint. Since the loading package is subject to a budget limitation, the approach considers both the probability that a part will be needed (its marginal product) and the cost which must be given up in order to include it in the package (its marginal cost). The composition of the package is then arranged according to the marginal analysis so as to obtain the maximum amount of supply protection from the available amount of budget.

In order to explain the approach, an example has been devised in which it is assumed that there are only six different kinds of items, which are included in the package. Item 'A' and 'B' from item group (1), item 'C' and 'D' out of item group (2), and, item 'E' and 'F' from item group (3). Each cost is different respectively. The budget limitation of the example loading package is 10 dollars. The objective is to select the composition of parts not exceeding the budget limit that will minimize the expected number of shortages. Table 9 shows each unit cost and each demand per month.

Table 9. **DEMAND - UNIT COST DATA FOR EXAMPLE**

Item	# Demands/mo.	Unit Cost($)	Items	# Demands/mo.	Unit Cost($)
A	0.0833	1.00	B	0.0833	2.00
C	1.0000	3.00	D	1.0000	4.00
E	2.5000	5.00	F	2.5000	6.00

The computations that are performed in order to obtain the optimal selection of items to go into the loading package. These are summerized in Table 10. A measure called "marginal protection" is computed for each possible unit of each of the six items, as shown in Table 10. This measures the additional product or value provided by each unit.

The first column of Table 10 means the unit of the particular item being considered. In other words, it indicates the first, second, third, and so on, unit of the particular item. The remaining columns of the Table 10 indicate for items, 'A', 'B', 'C', 'D', 'E', and F's probabilities that each of the units will be needed.

Table 10. PROBABILITY OF NEEDING INDICATED NUMBER OF UNITS

Unit Number (X)	Marginal Protection					
	Item A	Item B	Item C	Item D	Item E	Item F
1	.0799	.0799	.6321	.6321	.9178	.9178
2	.0033	.0033	.2642	.2642	.7127	.7127
3	.0001	.0001	.0803	.0803	.4562	.4562
4	.0000	.0000	.0190	.0190	.2424	.2424
5	0	0	.0037	.0037	.1088	.1088
6	0	0	.0006	.0006	.0420	.0420
7	0	0	.0001	.0001	.0142	.0142
8	0	0	.0000	.0000	.0043	.0043
9	0	0	0	0	.0012	.0012
10	0	0	0	0	.0003	.0003
11	0	0	0	0	.0002	.0002

In more detail, the probability that the first unit of 'A' will be needed during the month is 0.0799. The probability in 'C' is 0.6321, 0.9173 in 'E', and so on. This is not the probability that exactly one unit will be demanded, which is 0.0766, 0.3679, 0.2052, respectively (refer to Table 6). It is the probability that one or more will be demanded. In other words, the probability that the first unit will be needed is the probability of one demand (0.0766, see Table 6) plus the probability two demands (0.0032), plus the probability of three demands (0.0001).

The probability that the second unit of item 'A' will be needed is 0.0033, which is the probability that the demand will be two or more. The probability that the third unit of item 'A' will be demanded is 0.0001. The probability of need for each additional unit of 'C' and 'E' are computed in the same way from the corresponding probabilities of Table 10. These probabilities measure the marginal product or value of the particular units. Thus the second unit of 'C' is less than half as valuable as the first unit, since the probability of its being needed is 0.2642 compared with 0.6321 for the first unit. The third unit of item 'C' is a little over four times as valuable as the fourth unit. The first unit of 'D' is just as valuable as the first unit of 'C' because their demand rates are the same (i.e., 1.0). However, the first unit of 'E' is more valuable than the first unit of 'C'. Indeed, the second unit of 'E' is more valuable than the first unit of 'C'. While the first unit of item 'F' provides one and a half times as much protection as the first unit of item 'C', it costs two times as much ($6.00 as opposed to $3.00).

Consequently, it costs the package more in terms of dollars. To allow for the effect of dollar cost, the value obtained from each unit of each item is expressed on a "per dollar basis." In other words, the probability that each unit will be needed is divided by the unit cost of the particular item. In economic sense, this means that the marginal product is divided by the marginal cost. This provides what is called the "marginal protection per dollar", and this is shown for each of the six items in Table 11.

Table 11. **PRIORITY OF NEEDING INDICATED NUMBER OF UNITS (X)**

Unit Number (X)	Marginal Protection/Dollar					
	Item A	Item B	Item C	Item D	Item E	Item F
1	.07990	.03995	.21070	.15803	.18346	.15297
2	.00330	.00165	.08807	.06605	.14254	.11878
3	.00010	.00005	.02677	.02008	.09124	.07603
4	0	0	.00633	.00475	.04848	.04040
5	0	0	.00123	.00093	.02176	.01813
6	0	0	.00020	.00015	.00840	.00700
7	0	0	.00003	.00002	.00284	.00237
8	0	0	.00000	.00000	.00086	.00717
9	0	0	0	0	.00024	.00020
10	0	0	0	0	.00006	.00005
11	0	0	0	0	.00004	.00003

4. Selection of The Loading Package

The marginal protection per dollar for the first unit of item 'A' is 0.07990, which is the probability that one unit will be needed (0.0799) divided by the unit cost ($1.00). Although the probability that the first unit of 'B' will be needed is the same as for item 'A', but its marginal protection per dollar is only a half as much, because the unit cost of the item 'B' ($2.00) is two and an half times as much as that of the item 'A' ($1.00). The marginal protection of the first unit of item 'F' is one and a half times as much as the first unit of the item 'C', but the marginal protection per dollar of the first unit of item 'F' less than the first unit of item 'C'.

Now, what kinds of items do we select? How many items do we choose if we have already decided 'what kinds'? The process of selecting the units to go into the loading package is relatively simple, once the marginal protection per dollar has been computed. All of the units are arranged in descending value of marginal protection per dollar in Table 12.

Table 12. PRIORITY OF ITEM TO BE SELECTED ON MP/DOLLAR

Priority	MP/Dollar	Item No.	Unit Cost($)	Total Costs	Remark
1	.21070	C	3.00	3.00	*
2	.18346	E	5.00	8.00	*
3	.15803	D	4.00	12.00	.
4	.15297	F	6.00	18.00	.
5	.14254	E	5.00	23.00	.
6	.11878	F	6.00	29.00	.
7	.09124	E	5.00	34.00	.
8	.08807	C	3.00	37.00	.
9	.07990	A	1.00	38.00	.
10	.07603	F	6.00	44.00	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.

The first column shows the ranking of each unit of each item. The second column indicates its marginal protection per dollar. The third column identifies the item and suggests the unit of the item. The fourth column gives the unit cost of the item. The fifth column means the cumulative costs, which are the amount of cost that have been used up at any given cut-off point. The last column shows the mark '*' that indicates the inclusion of the package until the cut-off point occurs.

The first choice of a part for the loading package is the first unit of item 'C', which is a marginal protection per dollar of 0.21070. The second choice is the first unit of item 'E', which has a marginal protection per dollar of 0.18346. At this point, the loading package costs 8.00 dollars as can be seen in the fifth column. The third choice is the first unit of item 'D', at which point there are 12.00 dollars consumed for the loading package. However, at this point, the loading package costs 12 dollars, which is over the budget limits ($10.00). So, we stop. If the budget goes up more than 12 dollars, we could choose the first unit of item 'D'. Consequently, additional parts can be included until the budget limit has been reached. If the budget is 10.00 dollars as in the present example, the maximum amount of protection could be obtained by stocking 1 unit of item 'C', 1 unit of item 'E'. This choice would cost 8.00 dollars, which is within the budget limit. But 2.00 dollars of the budget are not used. In this case we do not use all of the budget available (30 dollars). This does not result in optimization of the budget. How can we do optimize the budget? If we solve by dynamic programming, we could optimize the budget.

5. Dynamic Programming

Dynamic programming is a quantitative analysis approach that is applicable to decision-making situations in which a series of interrelated decisions have to be made. This technique is based on the idea of dividing the problem into stages. The outcome of the decision at one stage affects the decision and outcome at next stage of the problem. Its use typically requires that a problem be broken down into a series of smaller subproblems that are solved sequentially [Ref. 15 : p. 638].

As a concept, dynamic programming is more flexible than most mathematical models and methods in management science. Prototype applications of dynamic programming have been reported to solve problems involving spare parts level determination, given space and weight constraints. It has been also used in a variety of production scheduling and numerous others [Ref. 16 : p. 605].

The dynamic programming solution approach involves four major steps, which in turn involve a number of important concepts.

- The overall problem is decomposed into subproblems called *stages*.
- The final stage of the problem is analyzed and solved for all of the possible conditions or states.
- Each preceding stage is solved, working backward from the final stage of the problem. This involves making an optimal *decision* for the intermediate stage being linked to its preceding stage by a *recursion relationship*. The recursion relationship that is employed involves a *return* at each stage of the problem. The return at a particular stage is the net benefit that accrues at that stage due to the decision selected and its interaction with the state of the system. The return can be thought of as the objective function of the dynamic programming problem.

- The initial stage of the problem is solved, and when this has been accomplished the optimal solution has been obtained.

We now deal with our present problem using dynamic programming. First, the problem is subdivided and analyzed by stages. In this problem there are six stages. Figure 2 shows it graphically.

The problem variables are defined as follows:

B = the budget

N = number of different types of items

n = index denoting items type n

D_n = decision variable at stage n

C_n = unit cost of item type n

R_n = return at stage n

S_n = state input to stage n

$f_n(S_{n,}D_n)$ = the return at stage n forS_n, D_n

$f_n^*(S_n)$ = the optimal return at stage $n$forS_n

$$= \underset{D_n}{\text{opt}}[R_n + f_{n-1}^*(S_{n-1})] = \underset{D_n}{\text{opt}}[R_n + f_{n-1}^*(S_n - D_n)]$$

Also, it is assumed that the problem is such that the solution values for D_n must be integers and the total return can be obtained as the sum of the individual returns, R_n. The problem is formulated as

$$\text{maximize} \quad \sum_{n=1}^{N} R_n = R_1 + R_2 + \ldots\ldots + R_N$$

subject to

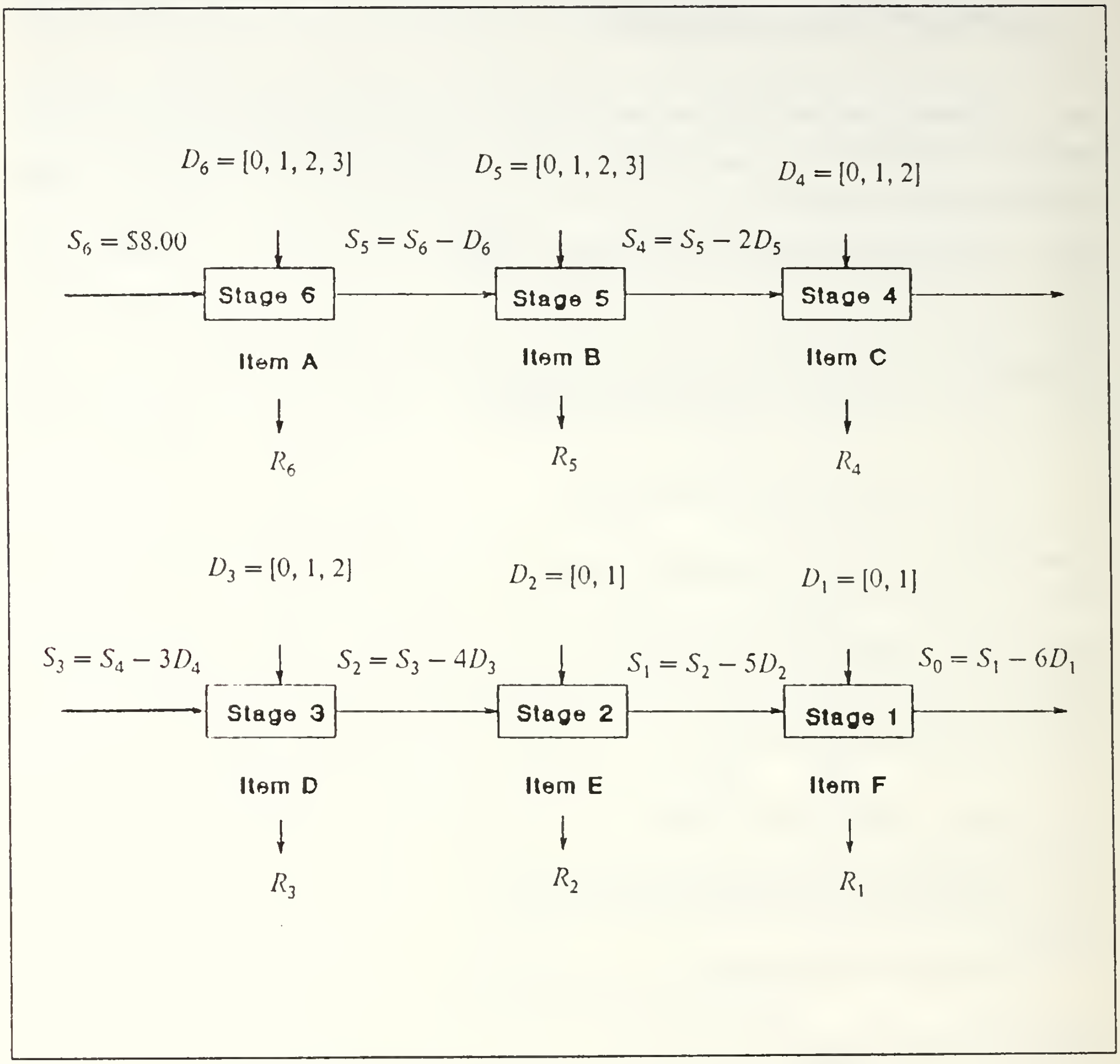

Figure 2. Graphical Summary of the Dynamic Programming Model

$$\sum_{n=1}^{N} C_n \times D_n \leq B$$

with $D_n = 0, 1, 2, \ldots..$

Decision (D_n)	Total Protection for Shortages (R_n)					
	Item A (S_1)	Item B (S_2)	Item C (S_3)	Item D (S_4)	Item E (S_5)	Item F (S_6)
1	.0799	.0799	.6321	.6321	0.9178	0.9178
2	.0832	.0832	.8963	.8963	1.6305	1.6305
3	.0833	.0833	.9766	.9766	2.0837	2.0837
4	.0833	.0833	.9956	.9956	2.3291	2.3291
5	.0833	.0833	.9993	.9993	2.4379	2.4379
6	.0833	.0833	.9999	.9999	2.4799	2.4799
7	.0833	.0833	1.0000	1.0000	2.4841	2.4841
8	.0833	.0833	1.0000	1.0000	2.4984	2.4984
9	.0833	.0833	1.0000	1.0000	2.4996	2.4996
10	.0833	.0833	1.0000	1.0000	2.4998	2.4998
11	.0833	.0833	1.0000	1.0000	2.5000	2.5000

The optimal solution is 1 unit of item C, 1 unit of item E, and 2 units of item A. If we have a budget of $8, the optimal solution is 1 unit of item C and 1 unit of item E. This is the same as the optimal solution obtained by marginal analysis. In any case, the budget is used up. This means that we can optimize the budget. (Refer to Appendix C for detail)

6. Comparison of Dynamic Programming and Marginal Analysis

Dynamic programming is a useful quantitative decision-making technique that has a very important role in sequential decision making. It affords a great deal of flexibility with respect to the variety of problems and cost functions that it allows to be modeled. Unfortunately, building the model is only part of the task, and solving the model using dynamic programming may present formidable problems.

Some dynamic programming problems suffer from what is known as the *curse of dimensionality*. This refers to the situation in which the dynamic programming formulation requires several states, which means that there are several state variables rather than the single state variable.

Although the inclusion of multiple states in a dynamic programming problem does not present a theoretical problem, it does cause significant computational difficulties. For dynamic programming problems in which the state variable is sional, the computational and data storage requirements increase dramatically. In the previous dynamic programming problem, we have to make $4 \times 4 \times 3 \times 3 \times 2 \times 2 = 576$ computations. If we have to select the loading packages with more than two constraints (states), we would meet more complex difficulties. However, in the marginal analysis case, we could solve it more easily because we can deal with the problem constraints separately. This illustrates the so-called *"curse of dimensionality,"* and accounts for the fact that most practical applications of dynamic programming have been limited to one or two state variables.

A second problem in using dynamic programming is the lack of computer software for solution of dynamic programming problems. Since each dynamic programming problem tends to be unique, there are no standard dynamic programming software packages available [Ref. 15 : p. 654].

While dynamic programming results in the optimization of the budget, marginal analysis does not always do. Our main point is that we have to optimize the budget as possible. Although dynamic programming method is more accurate than marginal analysis, it is more complicated and difficult to use, and needs for the user to be more skillful. However, even though marginal analysis does not always optimize the budget, it is very easy to apply because of its simpleness. Table 13 shows the comparison of the two methods in detail.

Table 13. CHARACTERISTICS OF THE TWO METHODS

Dynamic Programming	Marginal Analysis
Strong Points: - real optimization under a given budget - know the rank - more powerful Weak Points: - very complicated - need to be skillful	Strong Points: - makes possible to optimize the budget - easy to apply - know the rank easily - more flexible - easy to make computer program Weak Point: - not always optimize the budget

Why do we not always optimize the budget using marginal analysis? It is because there are only a few parts, and the unit costs are large in comparison to the total budget limit. In a real military problem involving thousands of items using a budget limit as large as 5000 dollars, which is a big budget relative to unit cost of any single item. it is possible to arrive at a selection of spares which costs within a few dollars of 5000 dollars.

Furthermore, we can develop the marginal analysis for more optimization of the budget by improving the procedure. If some of the budget is remained as unused, we could continually allocate the budget like follows. Suppose that B_R is the remaining budget, C_h is the unit cost of the highest ranking one which unit cost is less than equal B_R among the items unselected yet. If $B_R \leq C_h$, then allocate the budget to select the item.

In the previous marginal analysis problem, first we have to find out the highest ranking item which unit cost less than equal the remaining budget ($2.00) among items unselected. This means that we choose the next item to be included into the loading package between item 'A' and item 'B.' So, we select the first unit of item 'A' (Ranking = 9) (Refer to Table 12). One dollar is also remaining though. We do follow the same procedure again. Consequently, we allocate the last one dollar to the second unit of item 'A' (Ranking = 23). These solutions happen to be the optimal solutions in this case.

If there are huge items and relatively big budget, it is difficult to figure out the optimal solution on the previous procedure manually without an aid of a computer. For the purpose of this study, 200 sample parts of the APC line items with low-demand will be run with a computer program in Appendix B.

V. CONCLUSIONS

It is natural that the ROK Armed Forces should concentrate on more effective and efficient management of national defense resources. The ROK Army has to reserve the resources for war time economically. In war time an expected shortage of spares of a weapon system is an extremely critical factor in the Army's ability to fight.

For the initial stage of war, the ROK Army combat units currently reserve the spare parts for multi-item inventory system with predetermined "loading packages" based on the actual data. Especially for the newly developed and newly equipped weapon system, it is theoretically effective to apply marginal analysis to the development of the loading package in multi-item inventory systems with a constraint.

The package selected in the method described minimizes the expected number of shortages or demands that can not be satisfied by the package. Therefore, it represents the best way to allocate the limited resource (e.g., a budget dollar in this thesis) among the alternative parts to be included in the package. In this thesis, we treated each item as equally important to a weapon system. In other words, the essentiality of each item is the same. However, it is possible to extend the procedure to allow for the essentiality of the various shortages. The package is then designed so that the number of shortages weighted by their essentiality is minimized. This is done by treating the essentiality of a shortage as a probability value and multiplying the probabilities of demand for each unit of each item by the essentiality value of the item.

Therefore, it may be very helpful in order to minimize the expected shortages for "Team Spirit (TS)" exercise. [5] Each combat unit of the participating South Korean troops in the TS exercise usually has a mobility constraint. So, we can use the same procedure of the marginal analysis on the "marginal protection per weight unit" instead of the "marginal protection per cost unit".

In the preceding illustration, the Measure Of Effectiveness (MOE) is to minimize expected shortage. As an alternative, we could maximize another MOE for a fixed budget.

Finally, it may be useful to fix the MOE level desired, and to seek the minimum cost solution. For example, in some military loading packages, it may be desirable to fix the number of shortages, and determine the minimum budget. Marginal analysis can also solve this type of problem. In the present example, we want to exclude from the loading package those units of each item that contribute least (i.e., lowest marginal protection per dollar values), so that the expected shortages resulting from excluding these units equals the fixed MOE. The total budget of the units kept in the package then represents the minimum budget of the package for the fixed number of shortages.

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[5] This is a military exercise held annually by ROK-U.S. combined forces.

Finally, it is emphasized that this method is suggested not as a substitute for judgement but as an aid in applying it. Judgement will always be required because of the difficulty in assessing the exact essentiality of items in multi-item weapon systems.

APPENDIX A. REVIEW OF THREE INVENTORY MODELS

A. A DETERMINISTIC MODEL

This section will be devoted to a review of a deterministic inventory model. As we mentioned before, in the real world demands can almost never be predicted with certainty; instead they must be described in probabilistic terms. However, the deterministic model to be discussed is still of interest because it provides a simple framework for introducing the method of analysis that will be used in more complicated systems of real world problems. Furthermore, the results obtained from this model yield the proper sort of behavior qualitatively even when the deterministic demand assumption is removed.

Economic Order Quantity (EOQ)

The classical EOQ model is based on the following assumptions:

- The demand rate is known and constant.
- The lead time is known and constant.
- The entire lot size is added to inventory at the same time.
- No stockouts are permitted; since demand and lead time are known, stockouts can be avoided.
- The cost structure is fixed; order/setup costs are the same regardless of lot size, holding cost is a linear function based on average inventory, and no quantity discounts are given on large purchases.
- There is sufficient space, capacity, and capital to procure the desired quantity.
- The item is a single product; it does not interact with any other inventory items (there are no joint orders).

The size of an order that minimizes the total inventory cost is known as the economic order quantity(EOQ). The classical inventory model assumes the idealized situations shown in Figure 3, where Q is the order size. Upon receipt of an order, the inventory level is Q units. Units are withrawn from inventory at a constant demand rate, which is represented by the negative sloping lines. When the inventory reaches the reorder point R, a new order is placed for Q units. After a fixed time period, the order is received all at once and placed into inventory. The new lot is received just as the inventory level reaches zero, so the average inventory is $(Q+0)/2$ or $Q/2$. If stockouts are not permitted, the total inventory cost per year is graphically depicted by Figure 4 and by the following formula:

Total annual cost = (Purchase cost) + (Order cost) + (Holding cost)

$$TC = DP + \frac{DC}{Q} + \frac{QH}{2}$$

where

D = annual demand in units,

P = purchased cost of an item,

C = ordering cost,

H = PF = holding cost per unit per year,

Q = lot size or order quantity in units,

F = annual holding cost as a fraction of unit cost.

The total annual cost equation determines the annual purchase cost, which is the annual demand times the purchase cost per unit. The annual order cost is obtained as the number of orders per year (D/Q) times the cost to place an order (C). The annual holding cost is the average inventory (Q/2) times the annual unit holding cost (H). The sum of the three costs (purchase, order, and holding) is the total inventory cost per year for any given purchased item.

To obtain the EOQ, take the first derivative of total annual cost with respect to the lot size (Q) and set it equal to zero:

$$\frac{dTC}{dQ} = \frac{H}{2} - \frac{CD}{Q^2} = 0$$

Solving the equation for Q, we get the EOQ formula:

$$Q_0 = \frac{\sqrt{2CD}}{H} = \frac{\sqrt{2CD}}{PF} = EOQ$$

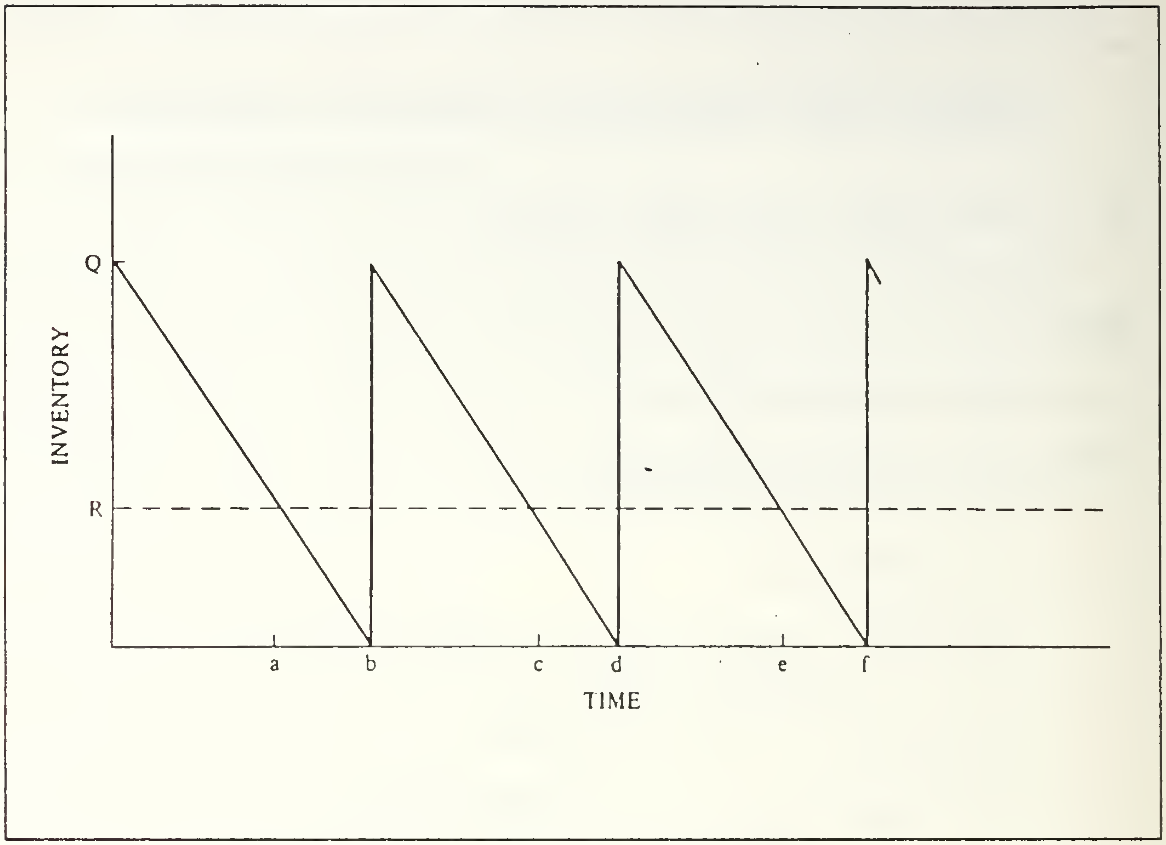

Figure 3. Classical Inventory Model.

The EOQ results in items with high unit cost being ordered frequently in small quantities ; items with low unit cost are ordered in large quantities. If the order cost, C is zero, orders are placed to satisfy each demand as it occurs, which results in no holding cost. If the holding cost, H is zero, only one order is placed for an amount that will satisfy the lifetime demand for the item.

Once the EOQ is known, the expected orders placed during the year, m and the average time between orders, T can be determined:

$$m = \frac{D}{Q_0} = \sqrt{\frac{HD}{2C}}$$

$$T = \frac{1}{m} = \frac{Q_0}{D} = \sqrt{\frac{2C}{DH}}$$

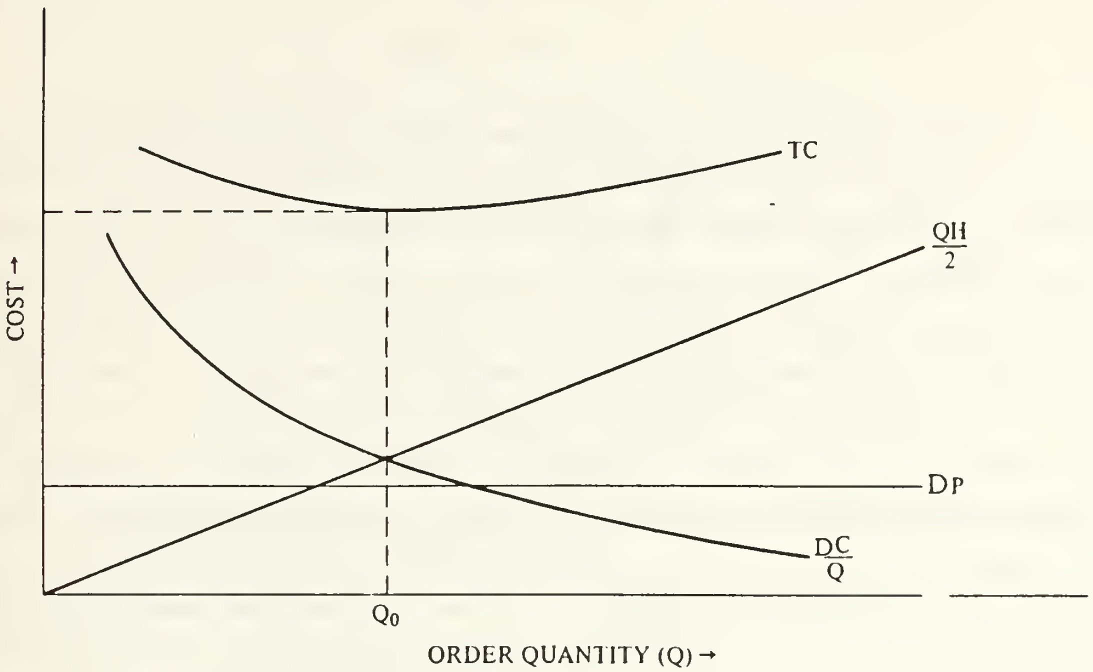

Figure 4. Annual Inventory Costs.

The order point is obtained by determining the demand that will occur during the lead time period. When the stock position (on hand + on order - backorders) reaches the reorder point, an order will be placed for Q_0 units, The reorder point, R is :

$$R = DL$$

where L is the lead time. If the lead time L is expressed in weeks, the reorder point is

$$R = \frac{DL}{52} \qquad (1 \text{ year} = 52 \text{ weeks})$$

The items are assumed to be received when the last item leaves the inventory, and the inventory level is restored to a level equal to the amount ordered. If the lead time is less than the average order interval, there will never be more than a single order outstanding. If the lead time is greater than the average order interval, there will always be at least one order outstanding. A simplified formula for the minimum total cost per year results:

$$TC = DP + HQ_{0.}$$

The EOQ minimizes the total cost function under a given set of circumstances. It is based on a given set of cost parameters, such as the order set up cost (C). If the assumptions are changed to allow stockouts, then the stockouts result in internal and external shortages. Internal shortages can result in lost production (idle men and machines) and a delay in a completion date (cost penalty). External shortages can result in backorder costs, present profit loss (lost sales), and future profit loss (goodwill erosion). In the military area, if stockouts occur there will be backorders, not "lost sales." The customer's (user's) reaction to a stockout condition can result in a backorder or a lost sale. With a backorder, the sale is not lost, but only delayed in shipment. Typically, a company will institute an emergency expending order to get the item, or the customer will be served from the next order of items to arrive. The backorder results in expediting costs, handling costs, and frequently extra shipping and packaging costs. If there were no costs associated with incurring backorders, no inventory would be held. If backorders were very expensive, they would never be allowed to occurred. However, there is an intermediate range of backordering costs where it is optimal to incur some backorders towards the end of an inventory cycle.

The backordering inventory model is shown in Figure 5. An order for Q units is placed when the stock on hand reaches the reorder point. The size of the stockout is $Q - V$ units, and the maximum inventory level is V units. The backordering cost per unit per year is K, and it is directly proportional to the length of the time delay. During time period t_3 one order is placed, so the order cost is C. The average holding cost during period t_1 is given as following:

$$H\frac{V}{2}t_1 = \frac{HV^2}{2R}$$

since

$$\frac{D}{1\text{ yr}} = \frac{V}{t_1} \; ; \qquad \text{then } \; t_1 = \frac{V}{D}$$

t_1 is the time period during which there is a positive inventory balance, and t_2 is the stockout time period. The average backordering cost during t_2 is as follows:

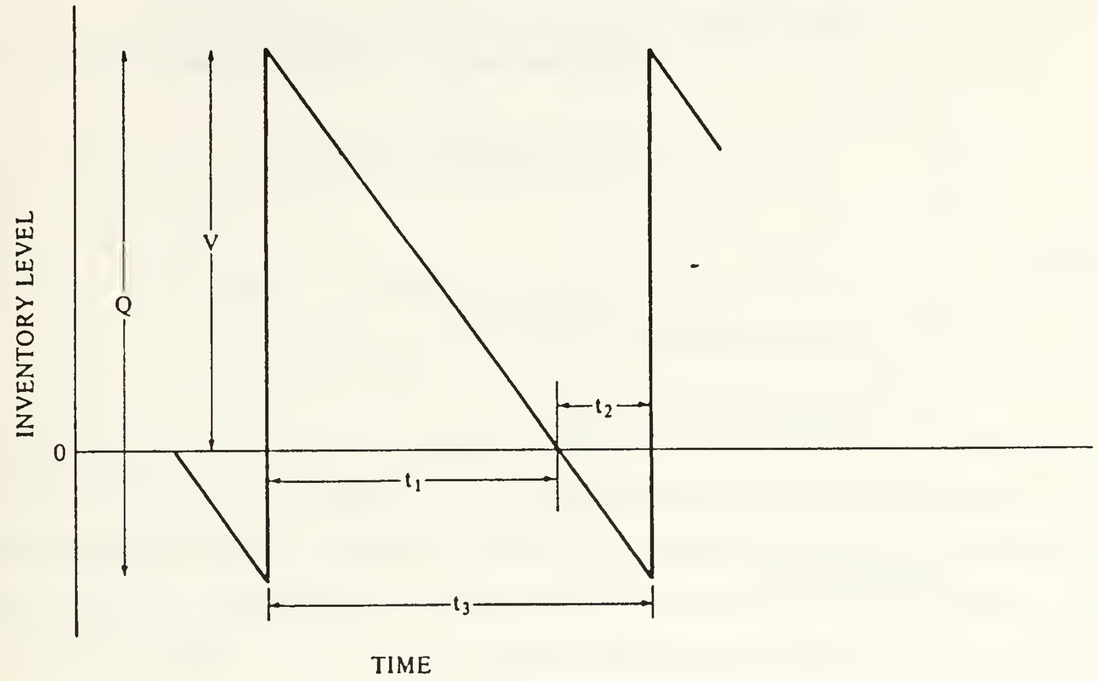

Figure 5. Backordering Inventory Model.

$$K\frac{(Q-V)t_2}{2} \quad = \quad K\frac{(Q-V)^2}{2D}$$

since

$$\frac{D}{1\ \text{yr}} = \frac{Q-V}{t_2}; \quad \text{then} \quad t_2 = \frac{Q-V}{D}$$

Therefore, the total cost for one time period of length t_3 is:

$$QP + C + \frac{HV^2}{2D} + K\frac{(Q-V)^2}{2D}$$

there are D/Q order periods of length t_3 in a year, so the total annual cost is obtained by multiplying the above equation by D/Q, which results in

Total annual cost = (Purchase cost) + (Order cost)

+ (Holding cost) + (Backorder cost)

$$TC = DP + C\frac{D}{Q} + \frac{HV^2}{2Q} + K\frac{(Q-V)^2}{2Q}$$

where

D = annual demand in units,

P = purchase cost of an item,

C = ordering cost per order,

Q = lot size,

H = holding cost per unit per year,

V = maximum inventory level,

K = backordering cost per unit per year,

D/Q = number orders per year.

To obtain optimal values for variable Q and V respectively, partial derivatives of the total annual cost with respect to Q and V are equated to zero.

$$Q^* = \sqrt{\frac{2DC}{H}}\sqrt{\frac{H+K}{K}},$$

$$V^* = \sqrt{\frac{2DC}{H}}\sqrt{\frac{K}{H+K}}.$$

When the backordering cost (K) is very large, $\sqrt{\frac{(H+K)}{K}}$ approaches one, and the economic order quantity corresponds to the classical case when there is no backorder, when the backordering cost (K) approaches zero, $\sqrt{K/(H+K)}$ approaches zero, and the maximum inventory level becomes zero. This is the situation where all items are backordered or handled on a special order basis, since V equals zero. The reorder point is calculated as

Reorder point = (Lead time demand) - (Backorders)

$$R = \frac{DL}{N} - (Q - V)$$

where

N = number of operating days per year,

L = lead time in days.

When an order not being placed until a certain number of backorders were obtained, the reorder point could be negative. Although the reorder point may be positive or negative with backordering, there will always be a period in which there is no stock available. When the backordering cost is finite, the reorder point will always be less than lead time demand.

B. A STOCHASTIC MODEL

It is likely that there is uncertainty in the demand for a given product or service, in the amount of time required for an order to be processed and delivered, and in the production time or time needed to provide a service. In fact, there is uncertainty associated with practically every aspect of a production and distribution system.

The basic EOQ model makes no allowance for uncertainty in the inputs to the inventory decision-making problem, and if inventory policy is sensitive to this uncertainty in the level of demand is a case in point. When demand is not known with certainty, it is usually economical to provide some level of safety stock as a hedge against unexpected demand that could deplete the inventory and cause a stock condition. How much safety stock should be added to the regular stock? How often will a stockout occur? What service level is provided? These are questions not answered by the basic EOQ model.

In developing an extended analysis, we need to recognize two types of inventory levels: the inventory level needed to meet the expected demand and the extra inventory needed for variations from the expected demand level. The objective is to find the fixed quantity that should be ordered and the minimum amount of stock that triggers the replacement order. However, the analysis will be based on the assumption that there is no interaction between the order quantity and the reorder point quantity. This approximate analysis will yield results that are reasonably close to the optimum inventory policy, and at the same time the higher order mathematics needed to find the optimum solution can be avoided.

Safety stock is determined directly from a forecast. Since forecasts are seldom exact, the safety stock protects against higher than expected demand levels. Safety stock has two effects on a firm's cost: it decreases the cost of stockouts, but it increases holding costs. Under the fixed order size system (Q-system), the reorder point R is composed of the mean lead time demand $\overline{M}$ plus the safety stock S.

In a realistic inventory system, as shown in Figure 6, the pattern of demand over time will be discrete and irregular. Figure 6 shows three cycles of an inventory system. In the first cycle, the demand during the lead time is so great that it results in a stockout. In the second cycle, the demand during the lead time is less than expected demand, and the replenishment is received is received before the safety stock is reached. In the third cycle, the demand during the lead time is greater than expected demand, but the safety stock is sufficient to absorb the demand.

Because forecasts are less than actual demand or suppliers sometimes fail to deliver goods on time, safety stocks are needed to protect against these two unfavorable contingencies:

- An actual demand is greater than forecasted.
- A late delivery of goods.

These unfavorable situations can result in a stockouts in the absence of safety stock. Each additional increase in safety stock provides decreasing benefits. The first unit of stock in excess of expected demand provides the largest increment of protection against stockout; the second unit provides less protection than the first unit, and so on. As the quantity of the safety stock is increased, the probability of a stockout decreases. The optimal level is the specific level in which the cost of holding additional units plus the expected stockout cost is minimized.

The probability of a stockout for a given item is simply the probability that the demand during the lead time will exceed the lead time demand. When discrete distributions are employed, the stockout probability is like the following:

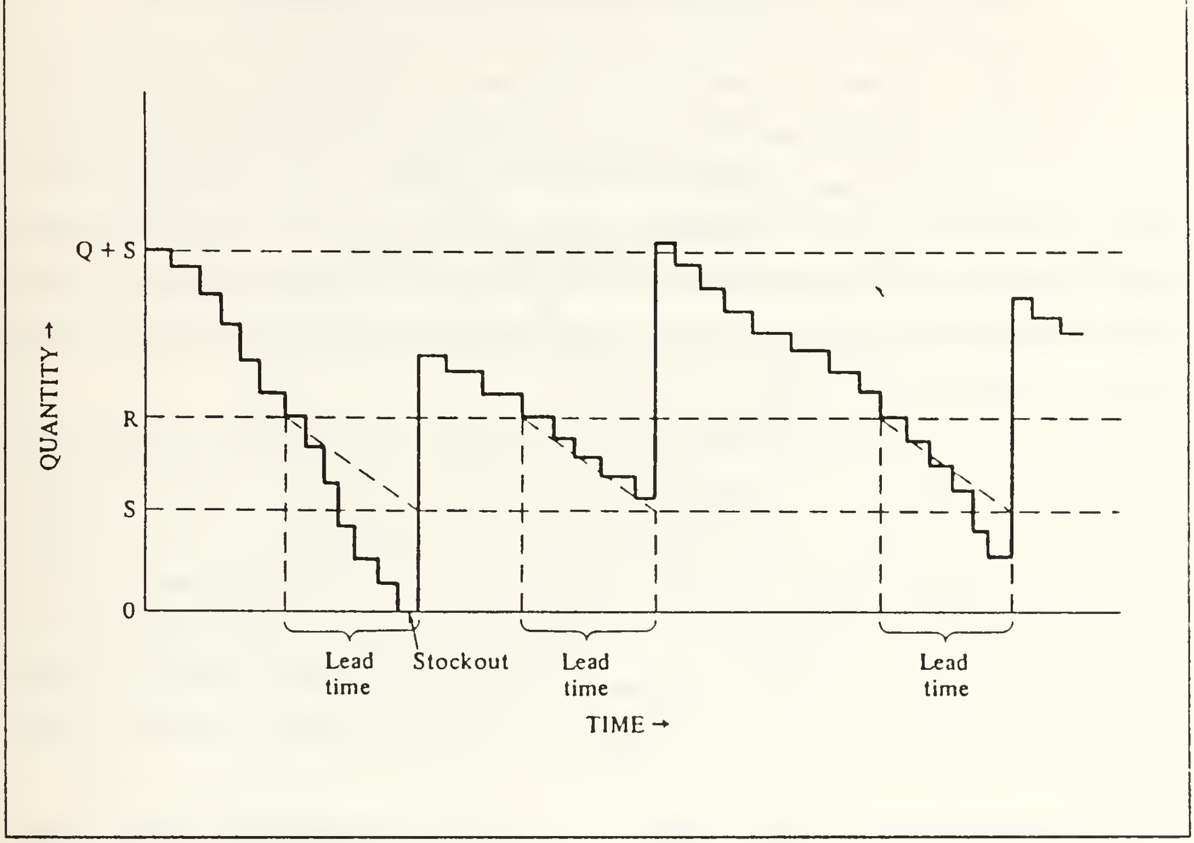

Figure 6. Realistic Inventory Model.

$$P(M > R) = \sum_{M=R+1}^{M_{\max}} P(M)$$

and the expected stockout quantity during the lead time is calculated as:

$$E(M > R) = \sum_{M=R+1}^{M_{\max}} (M - R)\, P(M)$$

where

$P(M > R) =$ probability of a stockout,

$R =$ demand during the lead time,

$P(M) =$ probability of demand M during the lead time,

$E(M > R) =$ expected stockout in units during the lead time.

The normal, Poisson, and negative exponential distributions have been found to be of considerable value in describing demand functions. The normal distribution has been found to describe many demand functions at the factory level; the Poisson, at the retail level; and the negative exponential, at the wholesale and retail levels. Of course, these distributions should not be automatically applied to any demand situation. Statistical tests should establish the basis for any standard distribution assumption concerning a demand function [Ref.6 : p. 191].

Stockout cost is usually the most difficult inventory cost to ascertain. Stockout cost may be due to backorders in the military area. It may be expressed on a per unit basis or a per stockout basis. If demand and lead time are constant, there will be no safety stock, since inventory decisions are made under certainty. Since there is perfect knowledge of demand and lead time, there is no problem. However, variable demand and constant lead time are frequently realistic for many items because contractual stipulations can render the lead time nearly certain. In this case, Figure 7 shows an illustration.

As mentioned above, in the military we only have to consider the backorder case. This is because customers (lower echelon) await the arrival of the order if a stockout occurs. If the probability of stockout is continuous and the stockout cost is on a per unit basis, the expected safety stock is defined as

$$S = \int_0^\infty (R - M) f(M) dM$$

$$= R \int_0^\infty f(M) dM - \int_0^\infty M f(M) dM$$

$$= R - \overline{M}$$

where

$\overline{M}$ = the expected lead time demand.

And the expected number of backorders per lead time is:

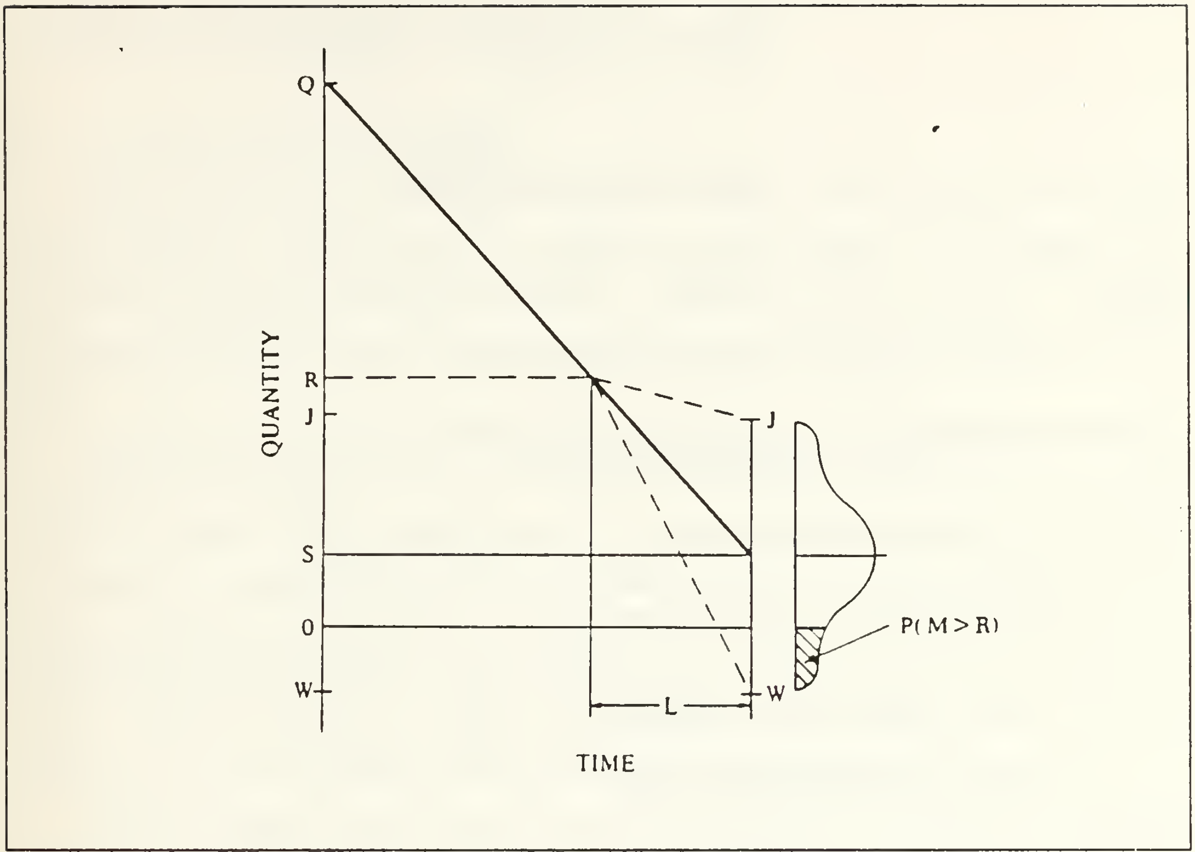

Figure 7. Variable Demand and Constant Lead Time.:

Notations:

R = reorder point Q = order quantity L = constant lead time

S = safety stock R-S = expected lead time demand

R-J = minimum lead time demand R-W = maximum lead time demand

P(M > R) = probability of stockout

$$E(M > R) = \int_R^{\infty} (M - R) f(M) dM$$

And annual safety stock cost is the holding cost plus the stockout cost:

$$TC_s = SH + \frac{AD}{Q} \int_R^{\infty} (M - R) f(M) dM$$

$$= H(R - \overline{M}) + \frac{AD}{Q} \int_{R}^{\infty} (M - R) f(M) dM$$

$$= H(R - \overline{M}) + AD \frac{E(M > R)}{Q}$$

where

TC_s = expected annual cost of safety stock,

$R = \overline{M} + S$ = reorder point in unit,

S = safety stock in units,

H = holding cost per unit per year,

A = backordering cost per unit,

D = annual demand in units,

Q = order quantity,

M = lead time demand in units (a random variable),

$\overline{M}$ = average lead time demand,

$f(M)$ = probability density function of lead time demand,

$M - R$ = size of stockout in units.

By taking the derivative of TC_s with respect to R and setting it equal to zero, the optimal probability of a stockout result in the following [Ref.6 : p. 196]:

$$P(M > R) = P(s) = \frac{HQ}{AD}$$

The above formula can be applied to both discrete and continuous probability distributions of lead time demand. When discrete distributions are employed, the exact optimum stockout probability is frequently unattainable because of the discrete nature of the data. When the optimum stockout probability cannot be attained, the next lower attainable stockout probability is selected [Ref. 6 : p.197.].

C. A SINGLE PERIOD MODEL

The essential characteristic of this model is that only a single time period, usually of finite length, is relevant and only a single procurement is made. This model is very well suited to demand that is non-continuous, changeable, and short-lived. This type problem is frequently referred to as the Christmas tree problem or the news-boy problem.

This model has a demand pattern with a limited sales (or usage) period. An item is ordered at the beginning of the period, and there is no opportunity for a second order during the period, since a second order would not arrive before the end of the period. Furthermore, there is insufficient time to replenish the stock to fill unsatisfied demand. If the demand during the period considered is greater than the order quantity, an opportunity-profit loss results. If the demand is less than the order quantity, the over stock is usually disposed of as spoilage or as obsolete, sold at a reduced price, or stored until the next period, with each of these alternatives incurring an associated cost.

When the demand is variable and the lead time is known, the single order inventory problem is in ascertaining the order size. If the demand is not known but a probability distribution of demand is available, the problem can be solved as decision-making under risk.

In the commercial sector, the order size that results in the largest expected profit or lowest expected cost is usually selected. We now show how to find the most profitable order quantity. The objective is to determine the order size that should be purchased at the beginning of the period to maximize the expected profit at the end of the period:

Expected profit (EP) = Expected revenue (ER) - Expected cost (EC) ;

(ER = Expected sales revenue + Expected salvage revenue)

$$ER = P_1[Q - \int_0^Q (Q - M)f(M)dM] + V\int_0^Q (Q - M)f(M)dM$$

$$= P_1Q + (V - P_1)\int_0^Q (Q - M)f(M)dM;$$

(EC = Purchase cost + Order cost + Expected sockout cost)

$$EC = PQ + C + A\int_{Q}^{\infty}(M - Q)f(M)dM,$$

$$EP = P_1Q + (V - P_1)\int_{0}^{Q}(Q - M)f(M)dM$$

$$- \mathrm{PQ} - C - A\int_{Q}^{\infty}(M - Q)f(M)dM$$

$$= P_1Q + (V - P_1)(Q - \overline{M}) - (P_1 - V + A)$$

$$\times\int_{Q}^{\infty}(M - Q)f(M)dM - PQ - C$$

where

$A =$ stockout cost per unit

$C =$ ordering cost,

$M =$ demand in units (a random variable),

$f(M) =$ probability density function of demand,

$M - Q =$ size of stockout in units,

$Q =$ single order quantity,

$Q - M =$ amount of excess inventory in units,

$P =$ unit purchase cost,

$P_1 =$ unit selling price,

$P(M > Q) =$ probability of stockout,

$V =$ salvage value per unit.

$\int_0^Q (Q - M) f(M) dM =$ expected number of excess units,

$\int_Q^\infty (M - Q) f(M) dM =$ expected stockout quantity in units,

To determine the maximum expected profit for a continuous distribution requires taking the derivative of the expected profit with respect to the order quantity and setting it equal to zero:

$$\frac{dEP}{dQ} = P_1 + V - P_1 + (P_1 + A - V)\, P(M > Q) - P = 0$$

Therefore, the optimum probability is as following:

$$P(M > Q) = P(s) = \frac{P - V}{P_1 + A - V} = \frac{\mathrm{ML}}{\mathrm{MP} + \mathrm{ML} + A}$$

where

$\mathrm{ML} = P - V =$ the marginal loss,

$\mathrm{MP} = P_1 - P =$ the marginal profit.

If the demand has a discrete distribution, the optimum probability of a stockout is not exactly attainable. If so, select the stock level with the next lower probability of a stockout [Ref. 6 : p. 308].

APPENDIX B. COMPUTER PROGRAM (FOTRAN) - MARGINAL ANALYSIS

```
        ---<<< INPUT DATA >>>---


----------------------------------------------------------
I  ITEM NO.  DEMANO  PRICE  I  ITEM NO.  DEMAND  PRICE  I
----------------------------------------------------------
I   xk-001    2.000  57.85  I   xk-002    0.660  27.85  I
I   xk-003    0.500  32.50  I   xk-004    1.250  78.00  I
I   xk-005    0.750  20.50  I   xk-006    0.083  10.00  I
I   xk-007    0.083  21.00  I   xk-008    1.000   2.00  I
I   xk-009    0.750  13.00  I   xk-010    0.150  17.50  I
I   xk-011    0.250  14.00  I   xk-012    2.330  20.00  I
I   xk-013    0.830   4.00  I   xk-014    1.580   3.00  I
I   xk-015    0.083   8.00  I   xk-016    0.580  17.00  I
I   xk-017    0.150  21.00  I   xk-018    1.670  42.00  I
I   xk-019    0.580   4.00  I   xk-020    0.083  22.00  I
I   XK-021    1.083  27.00  I   XK-022    0.150  18.00  I
I   XK-023    0.150  19.00  I   XK-024    0.330  17.00  I
I   XK-025    1.330  10.00  I   XK-026    0.250  12.00  I
I   XK-027    2.500  21.00  I   XK-028    1.170  12.00  I
I   XK-029    2.500  42.50  I   XK-030    0.083  20.00  I
I   XK-031    1.170  16.00  I   XK-032    1.420  23.45  I
I   XK-033    1.830  32.00  I   XK-034    2.420  40.50  I
I   XK-035    0.083   9.00  I   XK-036    2.000  30.45  I
I   XK-037    0.660  10.00  I   XK-038    0.660   4.00  I
I   XK-039    1.580  23.50  I   XK-040    0.250  18.50  I
I   XK-041    2.330  35.00  I   XK-042    0.083  11.00  I
I   XK-043    0.580  10.00  I   XK-044    1.670  22.00  I
I   XK-045    0.660   8.75  I   XK-046    0.750  11.00  I
I   XK-047    1.000   4.50  I   XK-048    0.080  34.25  I
I   XK-049    1.330  20.00  I   XK-050    0.250  20.50  I
I   XK-051    1.580  21.50  I   XK-052    0.150   8.00  I
I   XK-053    0.150  18.00  I   XK-054    0.580  21.00  I
I   XK-055    0.830  24.00  I   XK-056    2.080  42.00  I
I   XK-057    0.330   7.00  I   XK-058    0.083  12.00  I
I   XK-059    0.083 211.50  I   XK-060    1.830  12.00  I
I   XK-061    1.083  21.00  I   XK-062    0.150  15.00  I
I   XK-063    2.500   7.50  I   XK-064    1.000  22.00  I
I   XK-065    0.083   6.75  I   XK-066    1.750  20.00  I
I   XK-067    0.083   1.45  I   XK-068    2.500  40.32  I
I   XK-069    0.250   5.50  I   XK-070    0.400   4.00  I
I   XK-071    1.400 221.45  I   XK-072    0.500  12.00  I
I   XK-073    2.250  28.00  I   XK-074    0.660  42.00  I
I   XK-075    0.083 192.00  I   XK-076    0.080  34.00  I
I   XK-077    2.000  10.90  I   XK-078    2.400  28.25  I
I   XK-079    1.000  20.20  I   XK-080    0.250   3.75  I
I   XK-081    0.083  40.00  I   XK-082    0.083  19.00  I
I   XK-083    0.150  17.00  I   XK-084    2.250  20.00  I
I   XK-085    1.580  82.00  I   XK-086    0.250  90.75  I
I   XK-087    1.000  12.75  I   XK-088    1.750  40.00  I
I   XK-089    0.660  20.45  I   XK-090    2.500  24.00  I
I   XK-091    0.830  32.00  I   XK-092    0.083  90.00  I
I   XK-093    0.083  90.55  I   XK-094    1.830  42.15  I
I   XK-095    1.700  23.44  I   XK-096    0.150  25.50  I
I   XK-097    0.150  24.50  I   XK-098    1.900 117.00  I
I   XK-099    0.250  18.00  I   XK-100    1.000  91.00  I
```

```
I   XK-101      1.500  82.00  I   XK-102      0.083  72.00  I
I   XK-103      0.580 100.15  I   XK-104      0.400 111.00  I
I   XK-105      2.250  92.00  I   XK-106      0.083 102.00  I
I   XK-107      1.000  22.00  I   XK-108      0.250 100.00  I
I   XK-109      1.500  90.00  I   XK-110      0.500  25.00  I
I   XK-111      0.083 201.00  I   XK-112      2.250  31.45  I
I   XK-113      0.660 150.00  I   XK-114      2.500  42.00  I
I   XK-115      0.830  86.00  I   XK-116      0.083 111.00  I
I   XK-117      2.170  23.00  I   XK-118      1.170  42.00  I
I   XK-119      2.250  18.00  I   XK-120      1.500  14.00  I
I   XK-121      0.400  23.00  I   XK-122      2.500  56.00  I
I   XK-123      0.330  23.00  I   XK-124      1.000  54.00  I
I   XK-125      0.330  25.55  I   XK-126      0.400  20.00  I
I   XK-127      0.083  11.00  I   XK-128      1.250  42.00  I
I   XK-129      1.580  48.00  I   XK-130      0.330  22.00  I
I   XK-031      0.083   4.50  I   XK-132      0.083   4.00  I
I   XK-133      0.150  20.00  I   XK-134      0.150  23.50  I
I   XK-135      1.500  34.00  I   XK-136      0.830  11.75  I
I   XK-137      0.750  34.50  I   XK-138      1.910  43.00  I
I   XK-139      0.083   8.75  I   XK-140      0.083   3.00  I
I   XK-141      1.250 200.00  I   XK-142      0.400  42.00  I
I   XK-143      0.580  70.00  I   XK-144      0.150  18.00  I
I   XK-145      2.000  20.00  I   XK-146      2.500  42.00  I
I   XK-147      0.660  32.00  I   XK-148      0.660   8.00  I
I   XK-149      0.500  18.75  I   XK-150      2.420  20.00  I
I   XK-151      0.083  21.40  I   XK-152      0.150  42.00  I
I   XK-153      1.500  12.80  I   XK-154      0.500  21.50  I
I   XK-155      2.000  75.75  I   XK-156      1.170  40.00  I
I   XK-157      2.170  30.00  I   XK-158      0.580  80.00  I
I   XK-159      0.580  40.00  I   XK-160      0.250  21.75  I
I   XK-161      0.330  54.00  I   XK-162      0.580  25.00  I
I   XK-163      0.400  74.00  I   XK-164      0.150 108.42  I
I   XK-165      0.083  10.50  I   XK-166      0.500   2.50  I
I   XK-167      1.000  30.05  I   XK-168      0.500  10.25  I
I   XK-169      0.150   0.75  I   XK-170      2.500  33.50  I
I   XK-171      0.083  55.00  I   XK-172      0.083  12.00  I
I   XK-173      0.580  13.00  I   XK-174      2.500  10.00  I
I   XK-175      0.250  17.00  I   XK-176      0.400  12.10  I
I   XK-177      1.750  18.40  I   XK-178      0.250  17.90  I
I   XK-179      2.420  18.00  I   XK-180      0.330  22.45  I
I   XK-181      1.000  21.00  I   XK-182      0.580  17.00  I
I   XK-183      0.400  21.00  I   XK-184      2.500  12.55  I
I   XK-185      0.083  75.00  I   XK-186      0.083  92.00  I
I   XK-187      2.420  20.00  I   XK-188      0.150  20.50  I
I   XK-189      0.150  15.00  I   XK-190      0.330  10.50  I
I   XK-191      0.750  10.00  I   XK-192      0.400  82.75  I
I   XK-193      2.420  13.00  I   XK-194      1.580  10.00  I
I   XK-195      0.150  21.00  I   XK-196      0.150  70.10  I
I   XK-197      0.083  50.50  I   XK-198      1.830  82.00  I
I   XK-199      1.750  12.20  I   XK-200      2.500  20.00  I
-------------------------------------------------------------
```

```
      PROGRAM PARK
*********************************************************************
*                                                                   *
*     << DESCRIPTIONS FOR SOME VARIABLES >>                         *
*                                                                   *
*        DEMAND(N)  ; DEMANDS FOR EACH ITEM                         *
*        MPROT(N,M) ; MARGINAL PROTECTION (THIS MEANS THAT THE SUM OF*
*                     PROBABILITIES 'X' UNITS OR MORE DEMAND        *
*        MPD(N,M)   ; MARGINAL PROTECTION PER DOLLAR                *
*        BUDGET     ; BUDGET LIMIT                                  *
*        FPRTY(N*M) ; FINAL PRIORITY TO BUY                         *
*        ITEM(N)    ; ITEM NUMBER                                   *
*                                                                   *
*********************************************************************
      PARAMETER (N = 250, M = 8)
      REAL DEMAND(N), PRICE(N), WEIGHT(N), MPROT(N,M), MPD(N,M)
      REAL BUDGET, WGTLMT, ERR
      REAL FPRTY(N*M), FPRICE(N*M), FCOST(N*M)
      INTEGER X, NOITEM(N)
      CHARACTER*7 ITEM(N), FITEM(N*M)

      DATA BUDGET, WGTLMT, ERR / 2000., 1000., .0001 /
*
*     ---<< INPUT DATA >>---
*
      I = 0
   10 I = I + 1
      READ(5,*) ITEM(I),DEMAND(I),PRICE(I),WEIGHT(I)
      IF (ITEM(I) .EQ. 'NONE') GOTO 20
      GOTO 10
   20 NUMBER = I - 1

      WRITE(6,520)
      DO 30 I = 1, NUMBER, 2
           WRITE(6,550) (ITEM(J),DEMAND(J),PRICE(J), J = I, I+1)
   30 CONTINUE
      WRITE(6,560)

*
*     ---<< INITIALIZE MPROT(I,J) >>---
*
      DO 50 I = 1, N
           DO 40 J = 1, M
                MPROT(I,J) = 0.0
                MPD(I,J)   = 0.0
   40      CONTINUE
   50 CONTINUE

*
*     ---<< COMPUTE POISSON PROBABILITY FUNCTION , MARGINAL PROTECTION
*                                & MARGINAL PROTECTION PER DOLLAR >>---
*
      DO 70 K = 1, NUMBER
           IX  = 0
           J   = 0
           CDF = 0.
           TEMP1 = EXP(DEMAND(K))
   60      TEMP2 = TEMP1 * IFACT(IX)
           IF (IX .EQ. 0) THEN
                P = 1. / TEMP2
           ELSE
                P = (DEMAND(K) ** IX / TEMP2)
           ENDIF
           IX  = IX + 1
           J   = J + 1
           CDF = CDF + P
           MPROT(K,J) = 1. - CDF
           MPD(K,J)   = MPROT(K,J) / PRICE(K)
           IF (MPROT(K,J) .GE. ERR) GOTO 60
   70 CONTINUE
```

```
*     ---<< OUTPUT LIST # 1 >>---
*
      WRITE(6,600)
      WRITE(6,640)
      WRITE(6,610)
      WRITE(6,620) (I,I=0,7)
      WRITE(6,640)

      DO 90 I = 1, NUMBER
           WRITE(6,630) ITEM(I), DEMAND(I), (MPROT(I,J),J=1,M)
   90 CONTINUE
      WRITE(6,640)

*
*     ---<< OUTPUT LIST # 2 >>---
*
      WRITE(6,660)
      WRITE(6,640)
      WRITE(6,670)
      WRITE(6,620) (I,I=0,7)
      WRITE(6,640)
      DO 110 I = 1, NUMBER
           WRITE(6,630) ITEM(I), DEMAND(I), (MPD(I,J),J=1,M)
  110 CONTINUE
      WRITE(6,640)

*
*     ---<< FINDING PRIORITIES & TOTAL COSTS >>---
*
*
      MAXROW   = 0
      MAXCOL   = 0
      FCOST(1) = 0.

      DO 170 I = 1, NUMBER*M
           FPRTY(I) = -99.
           DO 150 K = 1, NUMBER
                DO 130 J = 1, M
                     IF (MPD(K,J) .GE. FPRTY(I)) THEN
                          MAXROW = K
                          MAXCOL = J
                          FPRTY(I)  = MPD(MAXROW, MAXCOL)
                     ENDIF
  130           CONTINUE
  150      CONTINUE

           FITEM(I)  = ITEM(MAXROW)
           FPRICE(I) = PRICE(MAXROW)
           IF (I .EQ. 1) THEN
                     FCOST(I) = FPRICE(I)
                ELSE
                     FCOST(I)  = FCOST(I-1) + FPRICE(I)
           ENDIF
           MPD(MAXROW, MAXCOL) = -99.
           IF (FCOST(I) .GT. BUDGET) THEN
                NUMBUD = I - 1
                GOTO 190
           ENDIF
  170 CONTINUE

*
*     ---<< OUTPUT LIST # 3 >>---
*
  190 WRITE(6,690)
      WRITE(6,740)
      WRITE(6,700)
      WRITE(6,740)
      DO 210 I = 1, NUMBUD
           WRITE(6,710) I, FPRTY(I), FITEM(I), FPRICE(I), FCOST(I)
  210 CONTINUE
      WRITE(6,740)
```

```
*
*     ---<< FINDING THE NUMBER OF EACH ITEMS CAN BE PURCHASED
*                                          WITH BUDGET LIMIT >>---
*
      DO 250 I = 1, NUMBER
           NOITEM(I) = 0
           DO 230 J = 1, NUMBUD
                IF(FITEM(J) .EQ. ITEM(I)) THEN
                     NOITEM(I) = NOITEM(I) + 1
                ENDIF
 230       CONTINUE
 250  CONTINUE

*
*     ---<< OUTPUT LIST # 4 >>---
*
      WRITE(6,730)
      DO 300 I = 1, NUMBER, 4
           WRITE(6,750) (ITEM(J), NOITEM(J),J = I, I+3)
 300  CONTINUE
      WRITE(6,650)

*
*     ---<< FORMAT LIST >>---
*
 500  FORMAT(I5,F10.3,F10.1,F5.0)
 520  FORMAT('1',5(/),10X,'---<<< INPUT DATA >>>---',5(/),5X,57('-'),
     *       /,5X,'I',2(2X,'ITEM NO.  DEMAND  PRICE  I'),/,5X,57('-'))
 550  FORMAT(5X,'I',2(3X,A7,2X,F6.3,1X,F6.2,'  I'))
 560  FORMAT(5X,57('-'))
 600  FORMAT('1',5(/),10X,'---<<< OUTPUT LIST  #1 >>>---',5(/))
 610  FORMAT(7X,'NO.',5X,'DEMAND',15X,'MARGINAL PROTECTION (N, X =< N)')
 620  FORMAT(/,27X,8(2X,I2,3X))
 630  FORMAT(' ',3X,A7,3X,F7.4,5X,8(F7.4))
 640  FORMAT(' ',85('-'))
 650  FORMAT(' ',5X,101('-'))
 660  FORMAT('1',/////,10X,'---<<< OUTPUT LIST  #2 >>>---',5(/))
 670  FORMAT(7X,'NO.',5X,'DEMAND',15X,'MARGINAL PROTECTION / $',
     *       ' (N, X =< N)')
 690  FORMAT('1',/////,10X,'---<<< OUTPUT LIST  #3 >>>---',5(/))
 700  FORMAT(7X,'PRIORITY',3X,'M.P. / $',4X,'ITEM NO.',3X,'UNIT COST',
     *       5X,'TOTAL')
 710  FORMAT(10X,I4,4X,F7.4,6X,A7,2X,F8.2,4X,F8.2)
 730  FORMAT('1',/////,10X,'---<<< LOADING PACKAGES >>>---',5(/),
     *       5X,101('-'),/,5X,'I',4(3X,'ITEM NO.   STOCKS    I'),/,
     *       5X,101('-'))
 740  FORMAT(' ',60('-'))
 750  FORMAT(5X,'I',4(4X,A7,4X,I3,6X,'I'))

      STOP
      END

      FUNCTION IFACT(IX)
      INTEGER IX, IFACT

      IFACT = 1
      IF (IX .GT. 0) THEN
           DO 10 I = 1, IX
                IFACT = IFACT * I
  10       CONTINUE
      ENDIF
      RETURN
      END
     ---<<< OUTPUT LIST  #1 >>>---


-------------------------------------------------------------------------------------
     NO.      DEMAND                MARGINAL PROTECTION (N, X => N)

                        N :   1      2      3      4      5      6      7      8
-------------------------------------------------------------------------------------
   xk-001     2.0000         0.8647 0.5940 0.3233 0.1429 0.0527 0.0166 0.0045 0.0011
   xk-002     0.6600         0.4831 0.1420 0.0295 0.0047 0.0006 0.0001 0.0000 0.0000
   xk-003     0.5000         0.3935 0.0902 0.0144 0.0018 0.0002 0.0000 0.0000 0.0000
   xk-004     1.2500         0.7135 0.3554 0.1315 0.0383 0.0091 0.0018 0.0003 0.0000
   xk-005     0.7500         0.5276 0.1734 0.0405 0.0073 0.0011 0.0001 0.0000 0.0000
   xk-006     0.0830         0.0796 0.0033 0.0001 0.0000 0.0000 0.0000 0.0000 0.0000
   xk-007     0.0830         0.0796 0.0033 0.0001 0.0000 0.0000 0.0000 0.0000 0.0000
   xk-008     1.0000         0.6321 0.2642 0.0803 0.0190 0.0037 0.0006 0.0001 0.0000
   xk-009     0.7500         0.5276 0.1734 0.0405 0.0073 0.0011 0.0001 0.0000 0.0000
   xk-010     0.1500         0.1393 0.0102 0.0005 0.0000 0.0000 0.0000 0.0000 0.0000
   xk-011     0.2500         0.2212 0.0265 0.0022 0.0001 0.0000 0.0000 0.0000 0.0000
   xk-012     2.3300         0.9027 0.6760 0.4119 0.2068 0.0873 0.0316 0.0100 0.0028
   xk-013     0.8300         0.5640 0.2020 0.0518 0.0103 0.0017 0.0002 0.0000 0.0000
   xk-014     1.5800         0.7940 0.4686 0.2115 0.0761 0.0226 0.0057 0.0012 0.0002
   xk-015     0.0830         0.0796 0.0033 0.0001 0.0000 0.0000 0.0000 0.0000 0.0000
   xk-016     0.5800         0.4401 0.1154 0.0212 0.0030 0.0003 0.0000 0.0000 0.0000
   xk-017     0.1500         0.1393 0.0102 0.0005 0.0000 0.0000 0.0000 0.0000 0.0000
   xk-018     1.6700         0.8118 0.4974 0.2349 0.0888 0.0277 0.0074 0.0017 0.0003
   xk-019     0.5800         0.4401 0.1154 0.0212 0.0030 0.0003 0.0000 0.0000 0.0000
   xk-020     0.0830         0.0796 0.0033 0.0001 0.0000 0.0000 0.0000 0.0000 0.0000
   XK-021     1.0830         0.6614 0.2947 0.0962 0.0245 0.0051 0.0009 0.0001 0.0000
   XK-022     0.1500         0.1393 0.0102 0.0005 0.0000 0.0000 0.0000 0.0000 0.0000
   XK-023     0.1500         0.1393 0.0102 0.0005 0.0000 0.0000 0.0000 0.0000 0.0000
   XK-024     0.3300         0.2811 0.0438 0.0047 0.0004 0.0000 0.0000 0.0000 0.0000
   XK-025     1.3300         0.7355 0.3838 0.1499 0.0461 0.0117 0.0025 0.0005 0.0001
   XK-026     0.2500         0.2212 0.0265 0.0022 0.0001 0.0000 0.0000 0.0000 0.0000
   XK-027     2.5000         0.9179 0.7127 0.4562 0.2424 0.1088 0.0420 0.0142 0.0042
   XK-028     1.1700         0.6896 0.3265 0.1141 0.0312 0.0070 0.0013 0.0002 0.0000
   XK-029     2.5000         0.9179 0.7127 0.4562 0.2424 0.1088 0.0420 0.0142 0.0042
   XK-030     0.0830         0.0796 0.0033 0.0001 0.0000 0.0000 0.0000 0.0000 0.0000
   XK-031     1.1700         0.6896 0.3265 0.1141 0.0312 0.0070 0.0013 0.0002 0.0000
   XK-032     1.4200         0.7583 0.4151 0.1714 0.0560 0.0151 0.0034 0.0007 0.0001
   XK-033     1.8300         0.8396 0.5460 0.2774 0.1136 0.0386 0.0112 0.0028 0.0006
   XK-034     2.4200         0.9111 0.6959 0.4355 0.2255 0.0984 0.0369 0.0121 0.0035
   XK-035     0.0830         0.0796 0.0033 0.0001 0.0000 0.0000 0.0000 0.0000 0.0000
   XK-036     2.0000         0.8647 0.5940 0.3233 0.1429 0.0527 0.0166 0.0045 0.0011
   XK-037     0.6600         0.4831 0.1420 0.0295 0.0047 0.0006 0.0001 0.0000 0.0000
   XK-038     0.6600         0.4831 0.1420 0.0295 0.0047 0.0006 0.0001 0.0000 0.0000
   XK-039     1.5800         0.7940 0.4686 0.2115 0.0761 0.0226 0.0057 0.0012 0.0002
   XK-040     0.2500         0.2212 0.0265 0.0022 0.0001 0.0000 0.0000 0.0000 0.0000
   XK-041     2.3300         0.9027 0.6760 0.4119 0.2068 0.0873 0.0316 0.0100 0.0028
   XK-042     0.0830         0.0796 0.0033 0.0001 0.0000 0.0000 0.0000 0.0000 0.0000
   XK-043     0.5800         0.4401 0.1154 0.0212 0.0030 0.0003 0.0000 0.0000 0.0000
   XK-044     1.6700         0.8118 0.4974 0.2349 0.0888 0.0277 0.0074 0.0017 0.0003
   XK-045     0.6600         0.4831 0.1420 0.0295 0.0047 0.0006 0.0001 0.0000 0.0000
   XK-046     0.7500         0.5276 0.1734 0.0405 0.0073 0.0011 0.0001 0.0000 0.0000
   XK-047     1.0000         0.6321 0.2642 0.0803 0.0190 0.0037 0.0006 0.0001 0.0000
   XK-048     0.0800         0.0769 0.0030 0.0001 0.0000 0.0000 0.0000 0.0000 0.0000
   XK-049     1.3300         0.7355 0.3838 0.1499 0.0461 0.0117 0.0025 0.0005 0.0001
   XK-050     0.2500         0.2212 0.0265 0.0022 0.0001 0.0000 0.0000 0.0000 0.0000
   XK-051     1.5800         0.7940 0.4686 0.2115 0.0761 0.0226 0.0057 0.0012 0.0002
   XK-052     0.1500         0.1393 0.0102 0.0005 0.0000 0.0000 0.0000 0.0000 0.0000
   XK-053     0.1500         0.1393 0.0102 0.0005 0.0000 0.0000 0.0000 0.0000 0.0000
   XK-054     0.5800         0.4401 0.1154 0.0212 0.0030 0.0003 0.0000 0.0000 0.0000
   XK-055     0.8300         0.5640 0.2020 0.0518 0.0103 0.0017 0.0002 0.0000 0.0000
   XK-056     2.0800         0.8751 0.6152 0.3450 0.1576 0.0602 0.0196 0.0056 0.0014
   XK-057     0.3300         0.2811 0.0438 0.0047 0.0004 0.0000 0.0000 0.0000 0.0000
   XK-058     0.0830         0.0796 0.0033 0.0001 0.0000 0.0000 0.0000 0.0000 0.0000
   XK-059     0.0830         0.0796 0.0033 0.0001 0.0000 0.0000 0.0000 0.0000 0.0000
   XK-060     1.8300         0.8396 0.5460 0.2774 0.1136 0.0386 0.0112 0.0028 0.0006
   XK-061     1.0830         0.6614 0.2947 0.0962 0.0245 0.0051 0.0009 0.0001 0.0000
   XK-062     0.1500         0.1393 0.0102 0.0005 0.0000 0.0000 0.0000 0.0000 0.0000
   XK-063     2.5000         0.9179 0.7127 0.4562 0.2424 0.1088 0.0420 0.0142 0.0042
```

XK-064	1.0000	0.6321	0.2642	0.0803	0.0190	0.0037	0.0006	0.0001	0.0000
XK-065	0.0830	0.0796	0.0033	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
XK-066	1.7500	0.8262	0.5221	0.2560	0.1008	0.0329	0.0091	0.0022	0.0005
XK-067	0.0830	0.0796	0.0033	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
XK-068	2.5000	0.9179	0.7127	0.4562	0.2424	0.1088	0.0420	0.0142	0.0042
XK-069	0.2500	0.2212	0.0265	0.0022	0.0001	0.0000	0.0000	0.0000	0.0000
XK-070	0.4000	0.3297	0.0616	0.0079	0.0008	0.0001	0.0000	0.0000	0.0000
XK-071	1.4000	0.7534	0.4082	0.1665	0.0537	0.0143	0.0032	0.0006	0.0001
XK-072	0.5000	0.3935	0.0902	0.0144	0.0018	0.0002	0.0000	0.0000	0.0000
XK-073	2.2500	0.8946	0.6575	0.3907	0.1906	0.0780	0.0274	0.0084	0.0023
XK-074	0.6600	0.4831	0.1420	0.0295	0.0047	0.0006	0.0001	0.0000	0.0000
XK-075	0.0830	0.0796	0.0033	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
XK-076	0.0800	0.0769	0.0030	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
XK-077	2.0000	0.8647	0.5940	0.3233	0.1429	0.0527	0.0166	0.0045	0.0011
XK-078	2.4000	0.9093	0.6916	0.4303	0.2213	0.0959	0.0357	0.0116	0.0033
XK-079	1.0000	0.6321	0.2642	0.0803	0.0190	0.0037	0.0006	0.0001	0.0000
XK-080	0.2500	0.2212	0.0265	0.0022	0.0001	0.0000	0.0000	0.0000	0.0000
XK-081	0.0830	0.0796	0.0033	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
XK-082	0.0830	0.0796	0.0033	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
XK-083	0.1500	0.1393	0.0102	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000
XK-084	2.2500	0.8946	0.6575	0.3907	0.1906	0.0780	0.0274	0.0084	0.0023
XK-085	1.5800	0.7940	0.4686	0.2115	0.0761	0.0226	0.0057	0.0012	0.0002
XK-086	0.2500	0.2212	0.0265	0.0022	0.0001	0.0000	0.0000	0.0000	0.0000
XK-087	1.0000	0.6321	0.2642	0.0803	0.0190	0.0037	0.0006	0.0001	0.0000
XK-088	1.7500	0.8262	0.5221	0.2560	0.1008	0.0329	0.0091	0.0022	0.0005
XK-089	0.6600	0.4831	0.1420	0.0295	0.0047	0.0006	0.0001	0.0000	0.0000
XK-090	2.5000	0.9179	0.7127	0.4562	0.2424	0.1088	0.0420	0.0142	0.0042
XK-091	0.8300	0.5640	0.2020	0.0518	0.0103	0.0017	0.0002	0.0000	0.0000
XK-092	0.0830	0.0796	0.0033	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
XK-093	0.0830	0.0796	0.0033	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
XK-094	1.8300	0.8396	0.5460	0.2774	0.1136	0.0386	0.0112	0.0028	0.0006
XK-095	1.7000	0.8173	0.5068	0.2428	0.0932	0.0296	0.0080	0.0019	0.0004
XK-096	0.1500	0.1393	0.0102	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000
XK-097	0.1500	0.1393	0.0102	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000
XK-098	1.9000	0.8504	0.5663	0.2963	0.1253	0.0441	0.0132	0.0034	0.0008
XK-099	0.2500	0.2212	0.0265	0.0022	0.0001	0.0000	0.0000	0.0000	0.0000
XK-100	1.0000	0.6321	0.2642	0.0803	0.0190	0.0037	0.0006	0.0001	0.0000
XK-101	1.5000	0.7769	0.4422	0.1912	0.0656	0.0186	0.0045	0.0009	0.0002
XK-102	0.0830	0.0796	0.0033	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
XK-103	0.5800	0.4401	0.1154	0.0212	0.0030	0.0003	0.0000	0.0000	0.0000
XK-104	0.4000	0.3297	0.0616	0.0079	0.0008	0.0001	0.0000	0.0000	0.0000
XK-105	2.2500	0.8946	0.6575	0.3907	0.1906	0.0780	0.0274	0.0084	0.0023
XK-106	0.0830	0.0796	0.0033	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
XK-107	1.0000	0.6321	0.2642	0.0803	0.0190	0.0037	0.0006	0.0001	0.0000
XK-108	0.2500	0.2212	0.0265	0.0022	0.0001	0.0000	0.0000	0.0000	0.0000
XK-109	1.5000	0.7769	0.4422	0.1912	0.0656	0.0186	0.0045	0.0009	0.0002
XK-110	0.5000	0.3935	0.0902	0.0144	0.0018	0.0002	0.0000	0.0000	0.0000
XK-111	0.0830	0.0796	0.0033	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
XK-112	2.2500	0.8946	0.6575	0.3907	0.1906	0.0780	0.0274	0.0084	0.0023
XK-113	0.6600	0.4831	0.1420	0.0295	0.0047	0.0006	0.0001	0.0000	0.0000
XK-114	2.5000	0.9179	0.7127	0.4562	0.2424	0.1088	0.0420	0.0142	0.0042
XK-115	0.8300	0.5640	0.2020	0.0518	0.0103	0.0017	0.0002	0.0000	0.0000
XK-116	0.0830	0.0796	0.0033	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
XK-117	2.1700	0.8858	0.6381	0.3692	0.1748	0.0693	0.0235	0.0070	0.0018
XK-118	1.1700	0.6896	0.3265	0.1141	0.0312	0.0070	0.0013	0.0002	0.0000
XK-119	2.2500	0.8946	0.6575	0.3907	0.1906	0.0780	0.0274	0.0084	0.0023
XK-120	1.5000	0.7769	0.4422	0.1912	0.0656	0.0186	0.0045	0.0009	0.0002
XK-121	0.4000	0.3297	0.0616	0.0079	0.0008	0.0001	0.0000	0.0000	0.0000
XK-122	2.5000	0.9179	0.7127	0.4562	0.2424	0.1088	0.0420	0.0142	0.0042
XK-123	0.3300	0.2811	0.0438	0.0047	0.0004	0.0000	0.0000	0.0000	0.0000
XK-124	1.0000	0.6321	0.2642	0.0803	0.0190	0.0037	0.0006	0.0001	0.0000
XK-125	0.3300	0.2811	0.0438	0.0047	0.0004	0.0000	0.0000	0.0000	0.0000
XK-126	0.4000	0.3297	0.0616	0.0079	0.0008	0.0001	0.0000	0.0000	0.0000
XK-127	0.0830	0.0796	0.0033	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
XK-128	1.2500	0.7135	0.3554	0.1315	0.0383	0.0091	0.0018	0.0003	0.0000
XK-129	1.5800	0.7940	0.4686	0.2115	0.0761	0.0226	0.0057	0.0012	0.0002
XK-130	0.3300	0.2811	0.0438	0.0047	0.0004	0.0000	0.0000	0.0000	0.0000
XK-031	0.0830	0.0796	0.0033	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
XK-132	0.0830	0.0796	0.0033	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000

XK-133	0.1500	0.1393	0.0102	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000
XK-134	0.1500	0.1393	0.0102	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000
XK-135	1.5000	0.7769	0.4422	0.1912	0.0656	0.0186	0.0045	0.0009	0.0002
XK-136	0.8300	0.5640	0.2020	0.0518	0.0103	0.0017	0.0002	0.0000	0.0000
XK-137	0.7500	0.5276	0.1734	0.0405	0.0073	0.0011	0.0001	0.0000	0.0000
XK-138	1.9100	0.8519	0.5691	0.2990	0.1270	0.0449	0.0135	0.0035	0.0008
XK-139	0.0830	0.0796	0.0033	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
XK-140	0.0830	0.0796	0.0033	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
XK-141	1.2500	0.7135	0.3554	0.1315	0.0383	0.0091	0.0018	0.0003	0.0000
XK-142	0.4000	0.3297	0.0616	0.0079	0.0008	0.0001	0.0000	0.0000	0.0000
XK-143	0.5800	0.4401	0.1154	0.0212	0.0030	0.0003	0.0000	0.0000	0.0000
XK-144	0.1500	0.1393	0.0102	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000
XK-145	2.0000	0.8647	0.5940	0.3233	0.1429	0.0527	0.0166	0.0045	0.0011
XK-146	2.5000	0.9179	0.7127	0.4562	0.2424	0.1088	0.0420	0.0142	0.0042
XK-147	0.6600	0.4831	0.1420	0.0295	0.0047	0.0006	0.0001	0.0000	0.0000
XK-148	0.6600	0.4831	0.1420	0.0295	0.0047	0.0006	0.0001	0.0000	0.0000
XK-149	0.5000	0.3935	0.0902	0.0144	0.0018	0.0002	0.0000	0.0000	0.0000
XK-150	2.4200	0.9111	0.6959	0.4355	0.2255	0.0984	0.0369	0.0121	0.0035
XK-151	0.0830	0.0796	0.0033	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
XK-152	0.1500	0.1393	0.0102	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000
XK-153	1.5000	0.7769	0.4422	0.1912	0.0656	0.0186	0.0045	0.0009	0.0002
XK-154	0.5000	0.3935	0.0902	0.0144	0.0018	0.0002	0.0000	0.0000	0.0000
XK-155	2.0000	0.8647	0.5940	0.3233	0.1429	0.0527	0.0166	0.0045	0.0011
XK-156	1.1700	0.6896	0.3265	0.1141	0.0312	0.0070	0.0013	0.0002	0.0000
XK-157	2.1700	0.8858	0.6381	0.3692	0.1748	0.0693	0.0235	0.0070	0.0018
XK-158	0.5800	0.4401	0.1154	0.0212	0.0030	0.0003	0.0000	0.0000	0.0000
XK-159	0.5800	0.4401	0.1154	0.0212	0.0030	0.0003	0.0000	0.0000	0.0000
XK-160	0.2500	0.2212	0.0265	0.0022	0.0001	0.0000	0.0000	0.0000	0.0000
XK-161	0.3300	0.2811	0.0438	0.0047	0.0004	0.0000	0.0000	0.0000	0.0000
XK-162	0.5800	0.4401	0.1154	0.0212	0.0030	0.0003	0.0000	0.0000	0.0000
XK-163	0.4000	0.3297	0.0616	0.0079	0.0008	0.0001	0.0000	0.0000	0.0000
XK-164	0.1500	0.1393	0.0102	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000
XK-165	0.0830	0.0796	0.0033	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
XK-166	0.5000	0.3935	0.0902	0.0144	0.0018	0.0002	0.0000	0.0000	0.0000
XK-167	1.0000	0.6321	0.2642	0.0803	0.0190	0.0037	0.0006	0.0001	0.0000
XK-168	0.5000	0.3935	0.0902	0.0144	0.0018	0.0002	0.0000	0.0000	0.0000
XK-169	0.1500	0.1393	0.0102	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000
XK-170	2.5000	0.9179	0.7127	0.4562	0.2424	0.1088	0.0420	0.0142	0.0042
XK-171	0.0830	0.0796	0.0033	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
XK-172	0.0830	0.0796	0.0033	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
XK-173	0.5800	0.4401	0.1154	0.0212	0.0030	0.0003	0.0000	0.0000	0.0000
XK-174	2.5000	0.9179	0.7127	0.4562	0.2424	0.1088	0.0420	0.0142	0.0042
XK-175	0.2500	0.2212	0.0265	0.0022	0.0001	0.0000	0.0000	0.0000	0.0000
XK-176	0.4000	0.3297	0.0616	0.0079	0.0008	0.0001	0.0000	0.0000	0.0000
XK-177	1.7500	0.8262	0.5221	0.2560	0.1008	0.0329	0.0091	0.0022	0.0005
XK-178	0.2500	0.2212	0.0265	0.0022	0.0001	0.0000	0.0000	0.0000	0.0000
XK-179	2.4200	0.9111	0.6959	0.4355	0.2255	0.0984	0.0369	0.0121	0.0035
XK-180	0.3300	0.2811	0.0438	0.0047	0.0004	0.0000	0.0000	0.0000	0.0000
XK-181	1.0000	0.6321	0.2642	0.0803	0.0190	0.0037	0.0006	0.0001	0.0000
XK-182	0.5800	0.4401	0.1154	0.0212	0.0030	0.0003	0.0000	0.0000	0.0000
XK-183	0.4000	0.3297	0.0616	0.0079	0.0008	0.0001	0.0000	0.0000	0.0000
XK-184	2.5000	0.9179	0.7127	0.4562	0.2424	0.1088	0.0420	0.0142	0.0042
XK-185	0.0830	0.0796	0.0033	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
XK-186	0.0830	0.0796	0.0033	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
XK-187	2.4200	0.9111	0.6959	0.4355	0.2255	0.0984	0.0369	0.0121	0.0035
XK-188	0.1500	0.1393	0.0102	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000
XK-189	0.1500	0.1393	0.0102	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000
XK-190	0.3300	0.2811	0.0438	0.0047	0.0004	0.0000	0.0000	0.0000	0.0000
XK-191	0.7500	0.5276	0.1734	0.0405	0.0073	0.0011	0.0001	0.0000	0.0000
XK-192	0.4000	0.3297	0.0616	0.0079	0.0008	0.0001	0.0000	0.0000	0.0000
XK-193	2.4200	0.9111	0.6959	0.4355	0.2255	0.0984	0.0369	0.0121	0.0035
XK-194	1.5800	0.7940	0.4686	0.2115	0.0761	0.0226	0.0057	0.0012	0.0002
XK-195	0.1500	0.1393	0.0102	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000
XK-196	0.1500	0.1393	0.0102	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000
XK-197	0.0830	0.0796	0.0033	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
XK-198	1.8300	0.8396	0.5460	0.2774	0.1136	0.0386	0.0112	0.0028	0.0006
XK-199	1.7500	0.8262	0.5221	0.2560	0.1008	0.0329	0.0091	0.0022	0.0005
XK-200	2.5000	0.9179	0.7127	0.4562	0.2424	0.1088	0.0420	0.0142	0.0042

```
---<<< OUTPUT LIST  #2 >>>---

------------------------------------------------------------------------------------------
   NO.       DEMAND                MARGINAL PROTECTION / $ (N, X => N)

                     N :     1      2      3      4      5      6      7      8
------------------------------------------------------------------------------------------
  xk-001     2.0000        0.0149 0.0103 0.0056 0.0025 0.0009 0.0003 0.0001 0.0000
  xk-002     0.6600        0.0173 0.0051 0.0011 0.0002 0.0000 0.0000 0.0000 0.0000
  xk-003     0.5000        0.0121 0.0028 0.0004 0.0001 0.0000 0.0000 0.0000 0.0000
  xk-004     1.2500        0.0091 0.0046 0.0017 0.0005 0.0001 0.0000 0.0000 0.0000
  xk-005     0.7500        0.0257 0.0085 0.0020 0.0004 0.0001 0.0000 0.0000 0.0000
  xk-006     0.0830        0.0080 0.0003 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
  xk-007     0.0830        0.0038 0.0002 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
  xk-008     1.0000        0.3161 0.1321 0.0402 0.0095 0.0018 0.0003 0.0000 0.0000
  xk-009     0.7500        0.0406 0.0133 0.0031 0.0006 0.0001 0.0000 0.0000 0.0000
  xk-010     0.1500        0.0080 0.0006 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
  xk-011     0.2500        0.0158 0.0019 0.0002 0.0000 0.0000 0.0000 0.0000 0.0000
  xk-012     2.3300        0.0451 0.0338 0.0206 0.0103 0.0044 0.0016 0.0005 0.0001
  xk-013     0.8300        0.1410 0.0505 0.0130 0.0026 0.0004 0.0001 0.0000 0.0000
  xk-014     1.5800        0.2647 0.1562 0.0705 0.0254 0.0075 0.0019 0.0004 0.0001
  xk-015     0.0830        0.0100 0.0004 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
  xk-016     0.5800        0.0259 0.0068 0.0012 0.0002 0.0000 0.0000 0.0000 0.0000
  xk-017     0.1500        0.0066 0.0005 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
  xk-018     1.6700        0.0193 0.0118 0.0056 0.0021 0.0007 0.0002 0.0000 0.0000
  xk-019     0.5800        0.1100 0.0288 0.0053 0.0007 0.0001 0.0000 0.0000 0.0000
  xk-020     0.0830        0.0036 0.0001 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
  XK-021     1.0830        0.0245 0.0109 0.0036 0.0009 0.0002 0.0000 0.0000 0.0000
  XK-022     0.1500        0.0077 0.0006 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
  XK-023     0.1500        0.0073 0.0005 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
  XK-024     0.3300        0.0165 0.0026 0.0003 0.0000 0.0000 0.0000 0.0000 0.0000
  XK-025     1.3300        0.0736 0.0384 0.0150 0.0046 0.0012 0.0002 0.0000 0.0000
  XK-026     0.2500        0.0184 0.0022 0.0002 0.0000 0.0000 0.0000 0.0000 0.0000
  XK-027     2.5000        0.0437 0.0339 0.0217 0.0115 0.0052 0.0020 0.0007 0.0002
  XK-028     1.1700        0.0575 0.0272 0.0095 0.0026 0.0006 0.0001 0.0000 0.0000
  XK-029     2.5000        0.0216 0.0168 0.0107 0.0057 0.0026 0.0010 0.0003 0.0001
  XK-030     0.0830        0.0040 0.0002 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
  XK-031     1.1700        0.0431 0.0204 0.0071 0.0020 0.0004 0.0001 0.0000 0.0000
  XK-032     1.4200        0.0323 0.0177 0.0073 0.0024 0.0006 0.0001 0.0000 0.0000
  XK-033     1.8300        0.0262 0.0171 0.0087 0.0035 0.0012 0.0003 0.0001 0.0000
  XK-034     2.4200        0.0225 0.0172 0.0108 0.0056 0.0024 0.0009 0.0003 0.0001
  XK-035     0.0830        0.0088 0.0004 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
  XK-036     2.0000        0.0284 0.0195 0.0106 0.0047 0.0017 0.0005 0.0001 0.0000
  XK-037     0.6600        0.0483 0.0142 0.0029 0.0005 0.0001 0.0000 0.0000 0.0000
  XK-038     0.6600        0.1208 0.0355 0.0074 0.0012 0.0002 0.0000 0.0000 0.0000
  XK-039     1.5800        0.0338 0.0199 0.0090 0.0032 0.0010 0.0002 0.0001 0.0000
  XK-040     0.2500        0.0120 0.0014 0.0001 0.0000 0.0000 0.0000 0.0000 0.0000
  XK-041     2.3300        0.0258 0.0193 0.0118 0.0059 0.0025 0.0009 0.0003 0.0001
  XK-042     0.0830        0.0072 0.0003 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
  XK-043     0.5800        0.0440 0.0115 0.0021 0.0003 0.0000 0.0000 0.0000 0.0000
  XK-044     1.6700        0.0369 0.0226 0.0107 0.0040 0.0013 0.0003 0.0001 0.0000
  XK-045     0.6600        0.0552 0.0162 0.0034 0.0005 0.0001 0.0000 0.0000 0.0000
  XK-046     0.7500        0.0480 0.0158 0.0037 0.0007 0.0001 0.0000 0.0000 0.0000
  XK-047     1.0000        0.1405 0.0587 0.0178 0.0042 0.0008 0.0001 0.0000 0.0000
  XK-048     0.0800        0.0022 0.0001 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
  XK-049     1.3300        0.0368 0.0192 0.0075 0.0023 0.0006 0.0001 0.0000 0.0000
  XK-050     0.2500        0.0108 0.0013 0.0001 0.0000 0.0000 0.0000 0.0000 0.0000
  XK-051     1.5800        0.0369 0.0218 0.0098 0.0035 0.0011 0.0003 0.0001 0.0000
  XK-052     0.1500        0.0174 0.0013 0.0001 0.0000 0.0000 0.0000 0.0000 0.0000
  XK-053     0.1500        0.0077 0.0006 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
  XK-054     0.5800        0.0210 0.0055 0.0010 0.0001 0.0000 0.0000 0.0000 0.0000
  XK-055     0.8300        0.0235 0.0084 0.0022 0.0004 0.0001 0.0000 0.0000 0.0000
  XK-056     2.0800        0.0208 0.0146 0.0082 0.0038 0.0014 0.0005 0.0001 0.0000
  XK-057     0.3300        0.0402 0.0063 0.0007 0.0001 0.0000 0.0000 0.0000 0.0000
  XK-058     0.0830        0.0066 0.0003 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
  XK-059     0.0830        0.0004 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
  XK-060     1.8300        0.0700 0.0455 0.0231 0.0095 0.0032 0.0009 0.0002 0.0001
  XK-061     1.0830        0.0315 0.0140 0.0046 0.0012 0.0002 0.0000 0.0000 0.0000
  XK-062     0.1500        0.0093 0.0007 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
  XK-063     2.5000        0.1224 0.0950 0.0608 0.0323 0.0145 0.0056 0.0019 0.0006
```

XK-064	1.0000	0.0287	0.0120	0.0037	0.0009	0.0002	0.0000	0.0000	0.0000
XK-065	0.0830	0.0118	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
XK-066	1.7500	0.0413	0.0261	0.0128	0.0050	0.0016	0.0005	0.0001	0.0000
XK-067	0.0830	0.0549	0.0022	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
XK-068	2.5000	0.0228	0.0177	0.0113	0.0060	0.0027	0.0010	0.0004	0.0001
XK-069	0.2500	0.0402	0.0048	0.0004	0.0000	0.0000	0.0000	0.0000	0.0000
XK-070	0.4000	0.0824	0.0154	0.0020	0.0002	0.0000	0.0000	0.0000	0.0000
XK-071	1.4000	0.0034	0.0018	0.0008	0.0002	0.0001	0.0000	0.0000	0.0000
XK-072	0.5000	0.0328	0.0075	0.0012	0.0001	0.0000	0.0000	0.0000	0.0000
XK-073	2.2500	0.0320	0.0235	0.0140	0.0068	0.0028	0.0010	0.0003	0.0001
XK-074	0.6600	0.0115	0.0034	0.0007	0.0001	0.0000	0.0000	0.0000	0.0000
XK-075	0.0830	0.0004	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
XK-076	0.0800	0.0023	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
XK-077	2.0000	0.0793	0.0545	0.0297	0.0131	0.0048	0.0015	0.0004	0.0001
XK-078	2.4000	0.0322	0.0245	0.0152	0.0078	0.0034	0.0013	0.0004	0.0001
XK-079	1.0000	0.0313	0.0131	0.0040	0.0009	0.0002	0.0000	0.0000	0.0000
XK-080	0.2500	0.0590	0.0071	0.0006	0.0000	0.0000	0.0000	0.0000	0.0000
XK-081	0.0830	0.0020	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
XK-082	0.0830	0.0042	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
XK-083	0.1500	0.0082	0.0006	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
XK-084	2.2500	0.0447	0.0329	0.0195	0.0095	0.0039	0.0014	0.0004	0.0001
XK-085	1.5800	0.0097	0.0057	0.0026	0.0009	0.0003	0.0001	0.0000	0.0000
XK-086	0.2500	0.0024	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
XK-087	1.0000	0.0496	0.0207	0.0063	0.0015	0.0003	0.0000	0.0000	0.0000
XK-088	1.7500	0.0207	0.0131	0.0064	0.0025	0.0008	0.0002	0.0001	0.0000
XK-089	0.6600	0.0236	0.0069	0.0014	0.0002	0.0000	0.0000	0.0000	0.0000
XK-090	2.5000	0.0382	0.0297	0.0190	0.0101	0.0045	0.0018	0.0006	0.0002
XK-091	0.8300	0.0176	0.0063	0.0016	0.0003	0.0001	0.0000	0.0000	0.0000
XK-092	0.0830	0.0009	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
XK-093	0.0830	0.0009	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
XK-094	1.8300	0.0199	0.0130	0.0066	0.0027	0.0009	0.0003	0.0001	0.0000
XK-095	1.7000	0.0349	0.0216	0.0104	0.0040	0.0013	0.0003	0.0001	0.0000
XK-096	0.1500	0.0055	0.0004	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
XK-097	0.1500	0.0057	0.0004	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
XK-098	1.9000	0.0073	0.0048	0.0025	0.0011	0.0004	0.0001	0.0000	0.0000
XK-099	0.2500	0.0123	0.0015	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
XK-100	1.0000	0.0069	0.0029	0.0009	0.0002	0.0000	0.0000	0.0000	0.0000
XK-101	1.5000	0.0095	0.0054	0.0023	0.0008	0.0002	0.0001	0.0000	0.0000
XK-102	0.0830	0.0011	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
XK-103	0.5800	0.0044	0.0012	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
XK-104	0.4000	0.0030	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
XK-105	2.2500	0.0097	0.0071	0.0042	0.0021	0.0008	0.0003	0.0001	0.0000
XK-106	0.0830	0.0008	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
XK-107	1.0000	0.0287	0.0120	0.0037	0.0009	0.0002	0.0000	0.0000	0.0000
XK-108	0.2500	0.0022	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
XK-109	1.5000	0.0086	0.0049	0.0021	0.0007	0.0002	0.0000	0.0000	0.0000
XK-110	0.5000	0.0157	0.0036	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000
XK-111	0.0830	0.0004	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
XK-112	2.2500	0.0284	0.0209	0.0124	0.0061	0.0025	0.0009	0.0003	0.0001
XK-113	0.6600	0.0032	0.0009	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
XK-114	2.5000	0.0219	0.0170	0.0109	0.0058	0.0026	0.0010	0.0003	0.0001
XK-115	0.8300	0.0066	0.0023	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000
XK-116	0.0830	0.0007	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
XK-117	2.1700	0.0385	0.0277	0.0161	0.0076	0.0030	0.0010	0.0003	0.0001
XK-118	1.1700	0.0164	0.0078	0.0027	0.0007	0.0002	0.0000	0.0000	0.0000
XK-119	2.2500	0.0497	0.0365	0.0217	0.0106	0.0043	0.0015	0.0005	0.0001
XK-120	1.5000	0.0555	0.0316	0.0137	0.0047	0.0013	0.0003	0.0001	0.0000
XK-121	0.4000	0.0143	0.0027	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000
XK-122	2.5000	0.0164	0.0127	0.0081	0.0043	0.0019	0.0008	0.0003	0.0001
XK-123	0.3300	0.0122	0.0019	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
XK-124	1.0000	0.0117	0.0049	0.0015	0.0004	0.0001	0.0000	0.0000	0.0000
XK-125	0.3300	0.0110	0.0017	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
XK-126	0.4000	0.0165	0.0031	0.0004	0.0000	0.0000	0.0000	0.0000	0.0000
XK-127	0.0830	0.0072	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
XK-128	1.2500	0.0170	0.0085	0.0031	0.0009	0.0002	0.0000	0.0000	0.0000
XK-129	1.5800	0.0165	0.0098	0.0044	0.0016	0.0005	0.0001	0.0000	0.0000
XK-130	0.3300	0.0128	0.0020	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
XK-031	0.0830	0.0177	0.0007	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
XK-132	0.0830	0.0199	0.0008	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

XK-133	0.1500	0.0070	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
XK-134	0.1500	0.0059	0.0004	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
XK-135	1.5000	0.0228	0.0130	0.0056	0.0019	0.0005	0.0001	0.0000	0.0000
XK-136	0.8300	0.0480	0.0172	0.0044	0.0009	0.0001	0.0000	0.0000	0.0000
XK-137	0.7500	0.0153	0.0050	0.0012	0.0002	0.0000	0.0000	0.0000	0.0000
XK-138	1.9100	0.0198	0.0132	0.0070	0.0030	0.0010	0.0003	0.0001	0.0000
XK-139	0.0830	0.0091	0.0004	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
XK-140	0.0830	0.0265	0.0011	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
XK-141	1.2500	0.0036	0.0018	0.0007	0.0002	0.0000	0.0000	0.0000	0.0000
XK-142	0.4000	0.0078	0.0015	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
XK-143	0.5800	0.0063	0.0016	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000
XK-144	0.1500	0.0077	0.0006	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
XK-145	2.0000	0.0432	0.0297	0.0162	0.0071	0.0026	0.0008	0.0002	0.0001
XK-146	2.5000	0.0219	0.0170	0.0109	0.0058	0.0026	0.0010	0.0003	0.0001
XK-147	0.6600	0.0151	0.0044	0.0009	0.0001	0.0000	0.0000	0.0000	0.0000
XK-148	0.6600	0.0604	0.0178	0.0037	0.0006	0.0001	0.0000	0.0000	0.0000
XK-149	0.5000	0.0210	0.0048	0.0008	0.0001	0.0000	0.0000	0.0000	0.0000
XK-150	2.4200	0.0456	0.0348	0.0218	0.0113	0.0049	0.0018	0.0006	0.0002
XK-151	0.0830	0.0037	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
XK-152	0.1500	0.0033	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
XK-153	1.5000	0.0607	0.0345	0.0149	0.0051	0.0015	0.0003	0.0001	0.0000
XK-154	0.5000	0.0183	0.0042	0.0007	0.0001	0.0000	0.0000	0.0000	0.0000
XK-155	2.0000	0.0114	0.0078	0.0043	0.0019	0.0007	0.0002	0.0001	0.0000
XK-156	1.1700	0.0172	0.0082	0.0029	0.0008	0.0002	0.0000	0.0000	0.0000
XK-157	2.1700	0.0295	0.0213	0.0123	0.0058	0.0023	0.0008	0.0002	0.0001
XK-158	0.5800	0.0055	0.0014	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000
XK-159	0.5800	0.0110	0.0029	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000
XK-160	0.2500	0.0102	0.0012	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
XK-161	0.3300	0.0052	0.0008	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
XK-162	0.5800	0.0176	0.0046	0.0008	0.0001	0.0000	0.0000	0.0000	0.0000
XK-163	0.4000	0.0045	0.0008	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
XK-164	0.1500	0.0013	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
XK-165	0.0830	0.0076	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
XK-166	0.5000	0.1574	0.0361	0.0058	0.0007	0.0001	0.0000	0.0000	0.0000
XK-167	1.0000	0.0210	0.0088	0.0027	0.0006	0.0001	0.0000	0.0000	0.0000
XK-168	0.5000	0.0384	0.0088	0.0014	0.0002	0.0000	0.0000	0.0000	0.0000
XK-169	0.1500	0.1857	0.0136	0.0007	0.0000	0.0000	0.0000	0.0000	0.0000
XK-170	2.5000	0.0274	0.0213	0.0136	0.0072	0.0032	0.0013	0.0004	0.0001
XK-171	0.0830	0.0014	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
XK-172	0.0830	0.0066	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
XK-173	0.5800	0.0339	0.0089	0.0016	0.0002	0.0000	0.0000	0.0000	0.0000
XK-174	2.5000	0.0918	0.0713	0.0456	0.0242	0.0109	0.0042	0.0014	0.0004
XK-175	0.2500	0.0130	0.0016	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
XK-176	0.4000	0.0272	0.0051	0.0007	0.0001	0.0000	0.0000	0.0000	0.0000
XK-177	1.7500	0.0449	0.0284	0.0139	0.0055	0.0018	0.0005	0.0001	0.0000
XK-178	0.2500	0.0124	0.0015	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
XK-179	2.4200	0.0506	0.0387	0.0242	0.0125	0.0055	0.0020	0.0007	0.0002
XK-180	0.3300	0.0125	0.0020	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
XK-181	1.0000	0.0301	0.0126	0.0038	0.0009	0.0002	0.0000	0.0000	0.0000
XK-182	0.5800	0.0259	0.0068	0.0012	0.0002	0.0000	0.0000	0.0000	0.0000
XK-183	0.4000	0.0157	0.0029	0.0004	0.0000	0.0000	0.0000	0.0000	0.0000
XK-184	2.5000	0.0731	0.0568	0.0363	0.0193	0.0087	0.0033	0.0011	0.0003
XK-185	0.0830	0.0011	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
XK-186	0.0830	0.0009	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
XK-187	2.4200	0.0456	0.0348	0.0218	0.0113	0.0049	0.0018	0.0006	0.0002
XK-188	0.1500	0.0068	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
XK-189	0.1500	0.0093	0.0007	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
XK-190	0.3300	0.0268	0.0042	0.0004	0.0000	0.0000	0.0000	0.0000	0.0000
XK-191	0.7500	0.0528	0.0173	0.0041	0.0007	0.0001	0.0000	0.0000	0.0000
XK-192	0.4000	0.0040	0.0007	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
XK-193	2.4200	0.0701	0.0535	0.0335	0.0173	0.0076	0.0028	0.0009	0.0003
XK-194	1.5800	0.0794	0.0469	0.0211	0.0076	0.0023	0.0006	0.0001	0.0000
XK-195	0.1500	0.0066	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
XK-196	0.1500	0.0020	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
XK-197	0.0830	0.0016	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
XK-198	1.8300	0.0102	0.0067	0.0034	0.0014	0.0005	0.0001	0.0000	0.0000
XK-199	1.7500	0.0677	0.0428	0.0210	0.0083	0.0027	0.0007	0.0002	0.0000
XK-200	2.5000	0.0459	0.0356	0.0228	0.0121	0.0054	0.0021	0.0007	0.0002

```
         ---<<< OUTPUT LIST  #3 >>>---

-----------------------------------------------------------
     PRIORITY   M.P. / $    ITEM NO.   UNIT COST     TOTAL
-----------------------------------------------------------
           1     0.3161      xk-008       2.00        2.00
           2     0.2647      xk-014       3.00        5.00
           3     0.1857      XK-169       0.75        5.75
           4     0.1574      XK-166       2.50        8.25
           5     0.1562      xk-014       3.00       11.25
           6     0.1410      xk-013       4.00       15.25
           7     0.1405      XK-047       4.50       19.75
           8     0.1321      xk-008       2.00       21.75
           9     0.1224      XK-063       7.50       29.25
          10     0.1208      XK-038       4.00       33.25
          11     0.1100      xk-019       4.00       37.25
          12     0.0950      XK-063       7.50       44.75
          13     0.0918      XK-174      10.00       54.75
          14     0.0824      XK-070       4.00       58.75
          15     0.0794      XK-194      10.00       68.75
          16     0.0793      XK-077      10.90       79.65
          17     0.0736      XK-025      10.00       89.65
          18     0.0731      XK-184      12.55      102.20
          19     0.0713      XK-174      10.00      112.20
          20     0.0705      xk-014       3.00      115.20
          21     0.0701      XK-193      13.00      128.20
          22     0.0700      XK-060      12.00      140.20
          23     0.0677      XK-199      12.20      152.40
          24     0.0608      XK-063       7.50      159.90
          25     0.0607      XK-153      12.80      172.70
          26     0.0604      XK-148       8.00      180.70
          27     0.0590      XK-080       3.75      184.45
          28     0.0587      XK-047       4.50      188.95
          29     0.0575      XK-028      12.00      200.95
          30     0.0568      XK-184      12.55      213.50
          31     0.0555      XK-120      14.00      227.50
          32     0.0552      XK-045       8.75      236.25
          33     0.0549      XK-067       1.45      237.70
          34     0.0545      XK-077      10.90      248.60
          35     0.0535      XK-193      13.00      261.60
          36     0.0528      XK-191      10.00      271.60
          37     0.0506      XK-179      18.00      289.60
          38     0.0505      xk-013       4.00      293.60
          39     0.0497      XK-119      18.00      311.60
          40     0.0496      XK-087      12.75      324.35
          41     0.0483      XK-037      10.00      334.35
          42     0.0480      XK-136      11.75      346.10
          43     0.0480      XK-046      11.00      357.10
          44     0.0469      XK-194      10.00      367.10
          45     0.0459      XK-200      20.00      387.10
          46     0.0456      XK-174      10.00      397.10
          47     0.0456      XK-187      20.00      417.10
          48     0.0456      XK-150      20.00      437.10
          49     0.0455      XK-060      12.00      449.10
          50     0.0451      xk-012      20.00      469.10
          51     0.0449      XK-177      18.40      487.50
          52     0.0447      XK-084      20.00      507.50
          53     0.0440      XK-043      10.00      517.50
          54     0.0437      XK-027      21.00      538.50
          55     0.0432      XK-145      20.00      558.50
          56     0.0431      XK-031      16.00      574.50
          57     0.0428      XK-199      12.20      586.70
          58     0.0413      XK-066      20.00      606.70
          59     0.0406      xk-009      13.00      619.70
          60     0.0402      XK-069       5.50      625.20
          61     0.0402      XK-057       7.00      632.20
          62     0.0402      xk-008       2.00      634.20
          63     0.0387      XK-179      18.00      652.20
```

64	0.0385	XK-117	23.00	675.20
65	0.0384	XK-168	10.25	685.45
66	0.0384	XK-025	10.00	695.45
67	0.0382	XK-090	24.00	719.45
68	0.0369	XK-051	21.50	740.95
69	0.0369	XK-044	22.00	762.95
70	0.0368	XK-049	20.00	782.95
71	0.0365	XK-119	18.00	800.95
72	0.0363	XK-184	12.55	813.50
73	0.0361	XK-166	2.50	816.00
74	0.0356	XK-200	20.00	836.00
75	0.0355	XK-038	4.00	840.00
76	0.0349	XK-095	23.44	863.44
77	0.0348	XK-187	20.00	883.44
78	0.0348	XK-150	20.00	903.44
79	0.0345	XK-153	12.80	916.24
80	0.0339	XK-027	21.00	937.24
81	0.0339	XK-173	13.00	950.24
82	0.0338	xk-012	20.00	970.24
83	0.0338	XK-039	23.50	993.74
84	0.0335	XK-193	13.00	1006.74
85	0.0329	XK-084	20.00	1026.74
86	0.0328	XK-072	12.00	1038.74
87	0.0323	XK-032	23.45	1062.19
88	0.0323	XK-063	7.50	1069.69
89	0.0322	XK-078	28.25	1097.94
90	0.0320	XK-073	28.00	1125.94
91	0.0316	XK-120	14.00	1139.94
92	0.0315	XK-061	21.00	1160.94
93	0.0313	XK-079	20.20	1181.14
94	0.0301	XK-181	21.00	1202.14
95	0.0297	XK-145	20.00	1222.14
96	0.0297	XK-090	24.00	1246.14
97	0.0297	XK-077	10.90	1257.04
98	0.0295	XK-157	30.00	1287.04
99	0.0288	xk-019	4.00	1291.04
100	0.0287	XK-107	22.00	1313.04
101	0.0287	XK-064	22.00	1335.04
102	0.0284	XK-112	31.45	1366.49
103	0.0284	XK-036	30.45	1396.94
104	0.0284	XK-177	18.40	1415.34
105	0.0277	XK-117	23.00	1438.34
106	0.0274	XK-170	33.50	1471.84
107	0.0272	XK-176	12.10	1483.94
108	0.0272	XK-028	12.00	1495.94
109	0.0268	XK-190	10.50	1506.44
110	0.0265	XK-140	3.00	1509.44
111	0.0262	XK-033	32.00	1541.44
112	0.0261	XK-066	20.00	1561.44
113	0.0259	XK-182	17.00	1578.44
114	0.0259	xk-016	17.00	1595.44
115	0.0258	XK-041	35.00	1630.44
116	0.0257	xk-005	20.50	1650.94
117	0.0254	xk-014	3.00	1653.94
118	0.0245	XK-021	27.00	1680.94
119	0.0245	XK-078	28.25	1709.19
120	0.0242	XK-174	10.00	1719.19
121	0.0242	XK-179	18.00	1737.19
122	0.0236	XK-089	20.45	1757.64
123	0.0235	XK-055	24.00	1781.64
124	0.0235	XK-073	28.00	1809.64
125	0.0231	XK-060	12.00	1821.64
126	0.0228	XK-135	34.00	1855.64
127	0.0228	XK-200	20.00	1875.64
128	0.0228	XK-068	40.32	1915.96
129	0.0226	XK-044	22.00	1937.96
130	0.0225	XK-034	40.50	1978.46

```
---<<< LOADING PACKAGES >>>---

-------------------------------------------------------------------------------
 ITEM NO.   STOCKS       ITEM NO.   STOCKS       ITEM NO.   STOCKS       ITEM NO.   STOCKS
-------------------------------------------------------------------------------
  xk-001       0          xk-002       0          xk-003       0          xk-004       0
  xk-005       1          xk-006       0          xk-007       0          xk-008       3
  xk-009       1          xk-010       0          xk-011       0          xk-012       2
  xk-013       2          xk-014       4          xk-015       0          xk-016       1
  xk-017       0          xk-018       0          xk-019       2          xk-020       0
  XK-021       1          XK-022       0          XK-023       0          XK-024       0
  XK-025       2          XK-026       0          XK-027       2          XK-028       2
  XK-029       0          XK-030       0          XK-031       1          XK-032       1
  XK-033       1          XK-034       1          XK-035       0          XK-036       1
  XK-037       1          XK-038       2          XK-039       1          XK-040       0
  XK-041       1          XK-042       0          XK-043       1          XK-044       2
  XK-045       1          XK-046       1          XK-047       2          XK-048       0
  XK-049       1          XK-050       0          XK-051       1          XK-052       0
  XK-053       0          XK-054       0          XK-055       1          XK-056       0
  XK-057       1          XK-058       0          XK-059       0          XK-060       3
  XK-061       1          XK-062       0          XK-063       4          XK-064       1
  XK-065       0          XK-066       2          XK-067       1          XK-068       1
  XK-069       1          XK-070       1          XK-071       0          XK-072       1
  XK-073       2          XK-074       0          XK-075       0          XK-076       0
  XK-077       3          XK-078       2          XK-079       1          XK-080       1
  XK-081       0          XK-082       0          XK-083       0          XK-084       2
  XK-085       0          XK-086       0          XK-087       1          XK-088       0
  XK-089       1          XK-090       2          XK-091       0          XK-092       0
  XK-093       0          XK-094       0          XK-095       1          XK-096       0
  XK-097       0          XK-098       0          XK-099       0          XK-100       0
  XK-101       0          XK-102       0          XK-103       0          XK-104       0
  XK-105       0          XK-106       0          XK-107       1          XK-108       0
  XK-109       0          XK-110       0          XK-111       0          XK-112       1
  XK-113       0          XK-114       0          XK-115       0          XK-116       0
  XK-117       2          XK-118       0          XK-119       2          XK-120       2
  XK-121       0          XK-122       0          XK-123       0          XK-124       0
  XK-125       0          XK-126       0          XK-127       0          XK-128       0
  XK-129       0          XK-130       0          XK-031       1          XK-132       0
  XK-133       0          XK-134       0          XK-135       1          XK-136       1
  XK-137       0          XK-138       0          XK-139       0          XK-140       1
  XK-141       0          XK-142       0          XK-143       0          XK-144       0
  XK-145       2          XK-146       0          XK-147       0          XK-148       1
  XK-149       0          XK-150       2          XK-151       0          XK-152       0
  XK-153       2          XK-154       0          XK-155       0          XK-156       0
  XK-157       1          XK-158       0          XK-159       0          XK-160       0
  XK-161       0          XK-162       0          XK-163       0          XK-164       0
  XK-165       0          XK-166       2          XK-167       0          XK-168       1
  XK-169       1          XK-170       1          XK-171       0          XK-172       0
  XK-173       1          XK-174       4          XK-175       0          XK-176       1
  XK-177       2          XK-178       0          XK-179       3          XK-180       0
  XK-181       1          XK-182       1          XK-183       0          XK-184       3
  XK-185       0          XK-186       0          XK-187       2          XK-188       0
  XK-189       0          XK-190       1          XK-191       1          XK-192       0
  XK-193       3          XK-194       2          XK-195       0          XK-196       0
  XK-197       0          XK-198       0          XK-199       2          XK-200       3
-------------------------------------------------------------------------------
```

APPENDIX C. PROCEDURE OF DYNAMIC PROGRAMMING

Formulation of Dynamic Programming

$$\text{Maximize} \quad \sum_{n=1}^{N} R_n = R_1 + R_2 + \ldots\ldots + R_N$$

subject to

$$\sum_{n=1}^{N} C_n \times D_n \leq B$$

with $\quad D_n = 0, 1, 2. \ldots..$

Decision (D_n)	Total Protection for Shortages (R_n)					
	Item A (S_1)	Item B (S_2)	Item C (S_3)	Item D (S_4)	Item E (S_5)	Item F (S_6)
1	.0799	.0799	.6321	.6321	0.9178	0.9178
2	.0832	.0832	.8963	.8963	1.6305	1.6305
3	.0833	.0833	.9766	.9766	2.0837	2.0837
4	.0833	.0833	.9956	.9956	2.3291	2.3291
5	.0833	.0833	.9993	.9993	2.4379	2.4379
6	.0833	.0833	.9999	.9999	2.4799	2.4799
7	.0833	.0833	1.0000	1.0000	2.4841	2.4841
8	.0833	.0833	1.0000	1.0000	2.4984	2.4984
9	.0833	.0833	1.0000	1.0000	2.4996	2.4996
10	.0833	.0833	1.0000	1.0000	2.4998	2.4998
11	.0833	.0833	1.0000	1.0000	2.5000	2.5000

The problem variables are defined as follows:

B = the budget

N = number of different types of items

n = index denoting items type n

D_n = decision variable at stage n

C_n = unit cost of item type n

R_n = return at stage n

S_n = state input to stage n

$f_n(S_n, D_n)$ = the return at stage n for S_n, D_n

$f_n^*(S_n)$ = the optimal return at stage n for S_n

$$= \underset{D_n}{\text{opt}}[R_n + f_{n-1}^*(S_{n-1})] = \underset{D_n}{\text{opt}}[R_n + f_{n-1}^*(S_n - D_n)]$$

STAGE 1 (ITEM 'F')

S_1	D_1	R_1
0 - 1^-	0	0 *
1 - 2^-	0	0 *
2 - 3^-	0	0 *
3 - 4^-	0	0 *
4 - 5^-	0	0 *
5 - 6^-	0	0 *
5 - 6^-	0	0 *
6 - 7^-	0 1	0. 0.9178 *
7 - 8^-	0 1	0. 0.9178 *
8	0 1	0. 0.9178 *
9	0 1	0. 0.9178 *
10	0 1	0. 0.9178 *

STAGE 2 (ITEM 'E')

S_2	D_2	R_2	S_1	$f_1^*(S_1)$	$f_2^*(S_2)$
$0 - 1^-$	0	0	$0 - 1^-$	0	0 *
$1 - 2^-$	0	0	$1 - 2^-$	0	0 *
$2 - 3^-$	0	0	$2 - 3^-$	0	0 *
$3 - 4^-$	0	0	$3 - 4^-$	0	0 *
$4 - 5^-$	0	0	$4 - 5^-$	0	0 *
$5 - 6^-$	0	0	$5 - 6^-$	0	0.
	1	0.9178	$0 - 1^-$	0	0.9178 *
$6 - 7^-$	0	0.	$6 - 7^-$	0.9178	0.9178
	1	0.9178	$1 - 2^-$	0	0.9178 *
$7 - 8^-$	0	0.	$7 - 8^-$	0.9178	0.9178
	1	0.9178	$2 - 3^-$	0	0.9178 *
8	0	0	8	0.9178	0.9178
	1	0.9178	3	0	0.9178 *
9	0	0	9	0.9178	0.9178
	1	0.9178	4	0	0.9178 *
10	0	0	10	0.9178	0.9178
	1	0.9178	5	0	0.9178
	2	1.6305	0	0	0.6305 *

STAGE 3 (ITEM 'D')

S_3	D_3	R_3	S_2	$f_2^*(S_2)$	$f_3^*(S_3)$
$0 - 1^-$	0	0	$0 - 1^-$	0	0 *
$1 - 2^-$	0	0	$1 - 2^-$	0	0 *
$2 - 3^-$	0	0	$2 - 3^-$	0	0 *
$3 - 4^-$	0	0	$3 - 4^-$	0	0 *
$4 - 5^-$	0 1	0 0.6321	$4 - 5^-$ $0 - 1^-$	0 0	0. 0.6321 *
$5 - 6^-$	0 1	0 0.6321	$5 - 6^-$ $0 - 1^-$	0.9178 0	0.9178 * 0.6321
$6 - 7^-$	0 1	0 0.6321	$6 - 7^-$ $2 - 3^-$	0.9178 0	0.9178 * 0.6321
$7 - 8^-$	0 1	0 0.6321	$7 - 8^-$ $3 - 4^-$	0.9178 0	0.9178 * 0.6321
8	0 1 2	0 0.6321 0.8963	8 4 0	0.9178 0 0	0.9178 * 0.6321 0.8963
9	0 1 2	0 0.6321 0.8963	9 5 1	0.9178 0.9178 0	0.9178 1.5499 * 0.8963
10	0 1 2	0 0.6321 0.8963	10 6 2	1.6305 0.9278 0	1.6305 * 1.5499 0.8963

STAGE 4 (ITEM 'C')

S_4	D_4	R_4	S_3	$f_3^*(S_3)$	$f_4^*(S_4)$
$0 - 1^-$	0	0	$0 - 1^-$	0	0 *
$1 - 2^-$	0	0	$1 - 2^-$	0	0 *
$2 - 3^-$	0	0	$2 - 3^-$	0	0 *
$3 - 4^-$	0	0	$3 - 4^-$	0	0.
	1	0.6321	$0 - 1^-$	0	0.6321 *
$4 - 5^-$	0	0	$4 - 5^-$	0.6321	0.6321
	1	0.6321	$1 - 2^-$	0	0.6321 *
$5 - 6^-$	0	0	$5 - 6^-$	0.9178	0.9178 *
	1	0.6321	$2 - 3^-$	0	0.6321
$6 - 7^-$	0	0	$6 - 7^-$	0.9178	0.9178 *
	1	0.6321	$3 - 4^-$	0	0.6321
	2	0.8963	$0 - 1^-$	0	0.8963
$7 - 8^-$	0	0	$7 - 8^-$	0.9178	0.9178
	1	0.6321	$4 - 5^-$	0.6321	1.2642 *
	2	0.8963	$1 - 2^-$	0	0.8963
8	0	0	8	0.9178	0.9178
	1	0.6321	5	0.9178	1.5499 *
	2	0.8963	2	0	0.8963
9	0	0.	9	1.5499	1.5499
	1	0.6321	6	0.9178	1.5499 *
	2	0.8963	3	0.	0.8963
	3	0.9766	0	0.	0.9766
10	0	0	10	1.6305	1.6305 *
	1	0.6321	7	0.9178	1.5499
	2	0.8963	4	0.6321	1.5284
	3	0.9766	1	0.	0.9766

STAGE 5 (ITEM 'B')

S_5	D_5	R_5	S_4	$f_4^*(S_4)$	$f_5^*(S_5)$
0 - 1^-	0	0	0 - 1^-	0	0 *
1 - 2^-	0	0	1 - 2^-	0	0 *
2 - 3^-	0	0	2 - 3^-	0	0.
	1	0.0799	0 - 1^-	0	0.0799 *
3 - 4^-	0	0	3 - 4^-	0.6321	0.6321 *
	1	0.0799	1 - 2^-	0	0.0799
4 - 5^-	0	0.	4 - 5^-	0.6321	0.6321 *
	1	0.0799	2 - 3^-	0	0.0799
	2	0.0832	0 - 1^-	0	0.0832
5 - 6^-	0	0	5 - 6^-	0.9178	0.9178 *
	1	0.0799	3 - 4^-	0.6321	0.7120
	2	0.0832	1 - 2^-	0	0.0832
6 - 7^-	0	0	6 - 7^-	0.9178	0.9178 *
	1	0.0799	4 - 5^-	0.6321	0.7120
	2	0.0832	2 - 3^-	0	0.0832
	3	0.0833	0 - 1^-	0	0.0833
7 - 8^-	0	0	7 - 8^-	1.2642	1.2642 *
	1	0.0799	5 - 6^-	0.9178	0.9977
	2	0.0832	3 - 4^-	0.6321	0.7153
	3	0.0833	1 - 2^-	0	0.0833
8	0	0	8	1.5499	1.5499 *
	1	0.0799	6	0.9178	0.9977
	2	0.0832	4	0.6321	0.7153
	3	0.0833	2	0	0.0833
9	0	0	9	1.5499	1.5499 *
	1	0.0799	7	1.2642	1.3441
	2	0.0832	5	0.9278	1.0010
	3	0.0833	3	0.6321	0.7154
10	0	0	10	1.6305	1.6305 *
	1	0.0799	8	1.5499	1.6298
	2	0.0832	6	0.9178	1.0010
	3	0.0833	4	0.6321	0.7154

STAGE 6 (ITEM 'A')

S_6	D_6	R_6	S_5	$f_5^*(S_5)$	$f_6^*(S_6)$
8	0	0	8	1.5499	1.5499 *
	1	0.0799	7	1.2642	1.3441
	2	0.0832	6	0.9178	1.0010
	3	0.0833	5	0.9178	1.0011
9	0	0	9	1.5499	1.5499
	1	0.0799	8	1.5499	1.6298 *
	2	0.0832	7	1.2642	1.3474
	3	0.0833	6	0.9178	0.0011
10	0	0	10	1.6305	1.6305
	1	0.0799	9	1.5499	1.6298
	2	0.0832	8	1.5499	1.6331 *
	3	0.0833	7	1.2642	1.3475

LIST OF REFERENCES

1. Huston, J. A., *The Sinews of War : Army Logistics,* Office of the Chief of Military History, U.S. Army, Washington D.C., 1966

2. Korean Institute of Defense Administration (KIDA), *The Analysis And Evaluation of the 'Yulgok' Plan,* Korean Army Headquaters, Seoul, Korea, Dec. 1986

3. Kwon, Tae-Young, "Defense Planning Management System", *ROK Army Management Journal* (Korean Version), Vol. Special, April 1985

4. ROK Army Deputy Chief of Staff, *Management Report* (Korean Version), ROK Army Pamphlet 1-59-Special, ROK Army Printing Office, April 1985

5. Ballou, R. H., *Business Logistics Management,* Prentice-Hall, Inc., 2nd edition, NJ, 1985

6. Tersine, R. J., *Principles of Inventory and Materials Management,* North-Holland, NY, 1988

7. Hadley, G., and Whitin, T. M., *Analysis of Inventory Systems,* Prentice-Hall, Englewood Cliff, NY, 1963

8. Enrick, N. L., *Inventory Management,* Chandler Publishing Company, S.F., CA, 1968

9. The Korean Army Logistics School, *The Korean Defense Industry* (Korean version), Seoul, Korea, 1986

10. Mansfield, Edwin, *Micro-economics,* W. W. Norton & Company, Inc., 5th edition, NY, 1985

11. Howard, G. T., *Minimizing Expected Shortages in a Multi-item Inventory System,* Naval Postgraduate Shcool, CA, January 1971

12. Hundai Heavy Industry Co., *The Analysis of Logistics for the XK-1* (Korean version), Chang Won, Korea, Jan. 1986

13. Van Matre, J. G., and Gilbreath, G. H., *Statistics for Business and Economics,* Business Publications, Inc., 3rd edition, TX, 1987

14. Davis, D. J., "An Analysis of Some Failure Data", *Journal of American Statistical Association,* Vol. 47, 1952

15. Markland, Robert E., and Sweigart, James R., *Quantitative Mathods,* John Wiley & Sons, Inc., NY, 1987

16. Lee, Sang M., Moore, Laurence J., and TaylorIII, Bernard W., *Management Science,* Second edition, Wm. C. Brown Publishers, IA, 1985

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